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Estimating fractal dimension using stable distributions and exploring long memory through ARFIMA models in Athens Stock Exchange

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It is argued that the study of the correct specification of returns distributions has attractive implications in financial economics. This study estimates Levy–stable (fractal) distributions that can accurately account for skewness, kurtosis, and fat tails. The Levy–stable family distributions are parametrized by the Levy index (α), $0 < \alpha \leq 2$, and include the normal distribution as a special case ($\alpha = 2$). The Levy index, α, is the fractal dimension of the probability space. The unique feature of Levy–stable family distributions is the existence of a relationship between the fractal dimension of the probability space and the fractal dimension of the time series. This relationship is simply expressed in terms of Hurst exponent (H), i.e. $\alpha = 1/H$. In addition, Hurst exponent is related to long-memory effects. Thus, estimating the Levy index allows us to determine long-memory effects. The immediate practical implication of the present work is that on the one hand we estimate the shape of returns distributions and on the other hand we investigate the fractal dimensions. Overall, then, the Levy–stable family distributions methodology appears to be useful for analysing the returns distribution, for understanding the fractal dimension of returns and for providing the researcher with direct insights into the long-memory effects of stock returns. A second approach to test the long memory hypothesis is attempted in this paper. This test involves an estimation of the ARFIMA models. A comparative analysis of the two approaches indicates the existence of long-memory in the Athens Stock Exchange. The results of this study are based on a sample of stocks from the Athens Stock Exchange using daily data.

I. INTRODUCTION

It is widely accepted that stock returns (1) follow a fat tailed non-normal distribution, (2) possess autocorrelations and partial autocorrelations that do not decay quickly to zero, and (3) seem to have non-periodic cycles. Evidence supporting (1), (2) and (3) is found in several empirical studies.

Normality is the exception rather than the rule in the stock markets. See Mandelbrot (1963), Fama (1965), Fielitz and Smith (1972), Blattberg and Gonedes (1974), Hsu *et al.* (1974), Simkowitz and Beedles (1980), Fielitz and Rozelle (1983), Kon (1984), Peters (1991), and Peters (1992).

In the case of the Athens Stock Exchange, Panas (1990) found non-normality of the distribution of stock returns. The behaviour of stock returns can be explained by the family of stable distributions. A number of articles have been written in recent as well as more distant times on the subject of stable distributions with characteristic exponent α between 1 and 2. Stable distributions are described by their characteristic function, which causes estimation and test procedures problems. Most studies, however, fail to provide statistical tests when the characteristic exponent ($= \alpha$) is different from two. In this paper, a statistical technique known as the bootstrap technique is employed to test the statistical significance of the characteristic exponent.

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The conventional linear methods fail to capture the behaviour of stock returns, described by (2) and (3). In such cases non-linear models such as ARCH (AutoRegressive Conditional Heteroscedastic), ARFIMA (AutoRegressive Fractionally Integrated Moving Average) and chaos are more appropriate.

Several authors (Panas and Stengos (1992), Koutmos *et al.* (1993), Papaioannou, G. I. and Philippatos, G. C. (1998) and Theodosiou *et al.* (1993)) have applied non-linear time series methods to study the dynamic processes of stock returns in the Athens Stock Exchange. Most recently, Barcoulas and Travlos (1998) have found empirical evidence that the time path of stock returns of the Athens Stock Exchange is consistent with nonlinear deterministic models (Chaos).

Another approach of interest deals with the estimation of stable distributions. Stable distributions are needed not only in the valuation of the distribution of stock returns but also to determine the memory of stock return processes. The unique feature of stable distributions is that they are fractal – see Peters (1994). For these stable distributions the characteristic exponent 'α' is equal to the inverse of the Hurst exponent (= H).

For $1/\alpha < \mathrm{H} < 1$, positive increments tend to be followed by positive increments (persistence); that is, we have a long-memory effect that occurs over multiple time series scales, whereas for $0 < \mathrm{H} < 1/\alpha$ positive increments tend to be followed by negative increments (antipersistence).

Thus, the estimates of α imply that knowledge of one characteristic parameter of a stable distribution gives us information about another classical exponent developed by Hurst (1951).

However, implementation of the above approach is based on the parameter of the stable distribution and it is valid only if the stock returns have a stable distribution. One technique that does not depend on knowledge of the underlying distribution is the ARFIMA models. ARFIMA models generalize linear ARIMA models by allowing for non integer differencing powers. Granger and Joyeux (1980) and Lo *et al.* (1988) proposed the use of ARFIMA procedures as long-memory models. A list of significant papers on long-memory analysis for stock returns includes Aydogan and Booth (1988), Lo (1991), Cheung *et al.* (1993), Cheung and Lai (1995) and Barkoulas and Baurn (1996).

The purpose of this paper is to present the estimates of the Hurst exponent, by estimating the stable distribution. This, in turn, will allow description of the distribution of stock returns and the fractal dimensionality of the underlying process. Although stable distributions and ARFIMA models are two completely different procedures, it is of interest to investigate whether they lead to the same conclusions regarding the behaviour of stock returns.

In addition, both stable distributions and ARFIMA models are tested using a sample of daily returns on thirteen stocks traded on the Athens Stock Exchange. The structure of the paper is as follows. Section II briefly describes the methodology. Section III contains the empirical results. The final section contains the summary and conclusions.

II. METHODOLOGICAL APPROACH

Let $Z_{it} = \log(\mathrm{P}_{it}/\mathrm{P}_{i,t-1})$ be the daily log price relatives. Before proceeding with the analysis the unit root test will be conducted to determine whether the underlying financial time series is a stationary process. To do that, the augmented Dickey–Fuller (ADF) (1979) and the Phillips–Perron (1988) methods will be used. The Dickey–Fuller test is based on the t-statistic associated with the p-coefficient in the following regression estimated by ordinary least squares:

$$\Delta y_t = \alpha + p y_{t-1} + \sum_{i=1}^{K} \Theta_i \Delta y_{t-1} + \varepsilon_t \tag{1}$$

where Δ is the difference operator.

k is selected as the lag order, to be large enough to ensure that the disturbances, ε_j, are serially uncorrelated disturbances. Subsequently the Phillips–Perron unit root test is performed. The Phillips–Perron test is based on the p-coefficient in the following regression estimated by ordinary least squares:

$$y_t = \mu + p y_{t-1} + \beta(t - \mathrm{T}/2) + e_t \tag{2}$$

where T is the number of observations. The hypothesis tested is

$$\mathrm{H}_0 : p = 1 \text{ versus } \mathrm{H}_1 : p < 1$$

The autocorrelation method has been used to investigate the dependency between the log of daily price relatives at time t and the log of daily price relatives at time $t + j$, where j refers to the time lag. We estimated the sample autocorrelation function (ACF) for Z_{it}, as well as the partial autocorrelation function (PACF).

In practice ACF and PACF will be estimated up to lag $n/4$, where n is the number of observations – see Box-Jenkins (1976). An overall test for serial correlation is carried out using the Ljung-Box Q-statistic. Under the null hypothesis, that all serial correlations are zero, $\mathrm{H}_0 : p_1 = p_2 = \cdots = p_j = 0$ the Ljung-Box Q statistic:

$$Q = n(n+2) \sum_{j=1}^{K} (n-j)^{-1} \cdot r_j^2 \tag{3}$$

where n = number of observations
r_j = sample ACF at lag j
k = the maximum lag considered $(= n/4)$

is distributed asymptotically as x^2 with k degrees of freedom.

A number of empirical studies have shown that the sample characteristics of log price returns are frequently inconsistent with those of a normal distribution.

Therefore, in order to overcome the shortcomings of normal distribution a more appropriate distribution is needed for log price returns.

The distributions used in this study have the property of being heavy-tailed (or fat-tailed). A distribution is heavy-tailed if $P[X > s] \sim s^{-\alpha}$, as $s \to \propto, 0 < \alpha < 2$, where $b(s) \sim g(s)$ means $b(s)/g(s) \to 1$ as $s \to \propto$. That is, regardless of the behaviour of the distribution for small values of the random variable, if the asymptotic shape of the distribution is hyperbolic, it is heavy-tailed. A family of heavy-tailed distributions is given by the Levy-stable distributions. The Levy-stable distributions have slowly decaying tails and infinite second moments. The approach taken here is to concentrate on the stable Levy distributions. These distributions have a much higher degree of variability, which makes them useful for modelling the empirical characteristics of log price returns. Most stable probability density distributions do not have closed analytical mathematical form. They are simply expressed in terms of their characteristic function which is the Fourier transform of the probability density function.

The logarithm of the characteristic function of a stable random variable X is given by:

$$\Phi(t) \equiv \log E\left(e^{ixt}\right) = i\delta t - \gamma|t|^{\alpha}\{1 + i\beta(t/|t|)\omega(|t|, \alpha)\}$$

where $i = \sqrt{-1}$; α is the Levy index $0 < \alpha \leq 2$ or a shape parameter (see Levy, 1937; Feller, 1971; Zolotarev, 1986); $\gamma(\gamma > 0)$ is a measure of dispersion; $\beta, -1 \leq \beta \leq 1$ is the skewness parameter; δ is a location parameter and

$$\omega(|t|, \alpha) = \begin{cases} \tan\left(\dfrac{\pi\alpha}{2}\right), & \text{if } \alpha \neq 1 \\ \dfrac{2}{\pi}\log|t|, & \text{if } \alpha = 1 \end{cases}$$

The Levy-stable distributions include the normal distribution as a special case ($\alpha = 2$). For the other possible values of Levy index (α), the stable distributions have slowly decaying tails and infinite second moments.

In addition, the Levy-stable distributions are fractal. Rescaled range (R/S) analysis gives a relationship between the Hurst exponent, H, and the Levy index (Peters, 1994). The relationship can be expressed as follows:

$$a = 1/\mathrm{H} \tag{4}$$

Equation 4 is used to calculate H, and the results are presented in the empirical section.

Thus, α measures the fractal dimension of the probability space. A strong statement is made by Peters (1994): 'The fractal dimension of the probability space is in this way related to the fractal dimension of the time series'. Thus, (i) the Hurst exponent of the time series is related to the Levy-index; (ii) the Levy-index ($\alpha = 1/\mathrm{H}$) is the fractal dimension of the probability space; (iii) 2-H is the fractal dimension of the time series; (iv) if $\mathrm{H} = 0.5$, the time-series exhibits a random walk; (v) if $1 \geq \mathrm{H} > 0.5$, we have a persistent time-series, (vi) if $0 \leq H < 0.5$, the time series is antipersistent, and (vii) if $\mathrm{H} = 1$, it corresponds to brownian motion.

Since most Levy-stable distributions do not have simple mathematical expressions, the main difficulty lies with the estimation of the parameters of Levy-stable distributions. The simplest method of parameter estimation is that of Fama and Roll (1968, 1971). The statistical properties, however, of the method are not known.

The method developed by Press (1972), which is based on a version of the method of moments, is used for α:

$$\hat{\alpha} = \frac{\log[\log|\Phi(t_1)|/\log|\Phi(t_2)|]}{\log(t_1/t_2)} \tag{5}$$

where t_1 and $t_2(t_1 \neq t_2)$ are two values of t and $\Phi(t)$ is the sample characteristic function.

One feasible way to estimate the standard error of $\hat{\alpha}$ is through a bootstrap procedure, see for instance Efron (1979). This study employs the bootstrap technique to derive the standard error of the parameter of interest ($\hat{\alpha}$). The obvious attraction of the bootstrap technique is that it may be applied to any statistic. Thus, even though the standard error of a statistic may be impossible to express in closed form, the bootstrap estimate can be really approximated. In the present case, interest is focused on determining the standard error of $\hat{\alpha}$. The basic idea is to create a large number ($K = 100$) of bootstrap samples to obtain the bootstrap replications: $\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_{100}$. The estimate of the standard error of $\hat{\alpha}$ is then given by:

$$\hat{\sigma}_{\hat{\alpha}} = \left\{\sum_{i=1}^{100} (\hat{\alpha}_i - \bar{\alpha})^2/99\right\}^{1/2} \tag{6}$$

where

$$\bar{\alpha} = \left(\sum_{i=1}^{100} \hat{\alpha}_i\right) \bigg/ 100$$

The analysis and the estimation of the Levy-stable distribution provide a way to test for fractal structure. However, in order to study the long-term dependence and non-periodic cycles and to understand better the low frequency dynamics of log price relatives a statistical model is needed that allows for fractal price dynamics.

Almost all early empirical studies of financial time-series focus on short memory processes – a stationary process which exhibits insignificant correlation beyond a small lag. The Box-Jenkins methodology or the ARMA model is used to investigate the short memory behaviour, a stationary process which exhibits insignificant correlation beyond a small lag of financial time-series, or following Brockwell and Davis (1991) it is said that a stationary process has short memory when its autocorrelation function is geometrically bounded. It is now recognized that in many cases, the financial time-series may exhibit the characteristics of long memory processes. A long memory process is a stationary process which exhibits significant correlation at large lags – see Beran (1994) and Fang *et al.* (1994). In other words, a stationary process has long memory if its autocorrelation function, say $\rho(h)$, has a hyperbolic decay, $\rho(h) \sim Ch^{2d-1}$ as $h \rightarrow \propto$ where $C \neq 0$ and $d < 0.5$ – see Brockwell and Davis (1991). Thus the autocorrelation function of a long memory process follows a power law, as compared to the exponential decay. Power-law decay is slower than the exponential decay and, since $d < 0.5$, the sum of the autocorrelation coefficients of such a series approaches infinity. The speed of decay of the series autocorrelation function is related to the Hurst exponent by $\mathrm{H} = 0.5 + d$. Thus, the next step in our methodology is to investigate whether a short or long-memory process works best as a model for the stationary financial time series under study.

The Autoregressive Fractally Integrated Moving Average (ARFIMA) model provides an appropriate framework – Mandelbrot (1977), Granger and Joyeux (1980), Hosking (1981), Lo (1991) – for analysis of the long-term dependence structure of a time series.

A time series $\{X_t\}$ follows an ARFIMA (p,d,q) process if:

$$\Phi_p(\mathrm{B})(1-\mathrm{B})^d x_t = \Theta_q(\mathrm{B})\alpha_t \tag{7}$$

where

$$\Phi_p(\mathrm{B}) = 1 - \phi_1\mathrm{B} - \cdots - \phi_p\mathrm{B}^p$$

$$\Theta_q(B) = 1 + \Theta_1 B + \cdots + \Theta_q B^q$$

$(1-B)^d = 1 - dB - \dfrac{d}{2}(1-d)B^2 - \cdots$ and $\{\alpha_t\}$ are i.i.d. disturbances with mean zero and variance $\sigma_\alpha^2 < \propto$.

The properties of the ARFIMA model are presented by Granger and Joyeux (1980) and in Hosking (1981): (p_1): if the roots of $\Phi_p(\mathrm{B})$ and $\Theta_q(\mathrm{B})$ are outside the unit circle and $d < |0.5|$, then X_t is both stationary and invertible; (p_2): if $0 < d < 0.5$ the ARFIMA model is capable of generating stationary series which are persistent. In this case the process displays long memory characteristics, such as a hyperbolic autocorrelation decay to zero; (p_3): if $d \geq 0.5$ the process is non-stationary; (p_4): when $d = 0$ there is an ARMA process and it exhibits short memory; (p_5): when $-0.5 < d < 0$ the ARFIMA process is said to exhibit intermediate memory or antipersistence.

Geweke and Porter-Hudak (1983) propose a non-parametric procedure to obtain an estimate of the fractional difference parameter *d*. They propose the following periodogram or spectrum regression:

$$\ln\{I_x(\omega_\lambda)\} = \beta_0 - d \cdot \ln\{4\sin^2(\omega_\lambda/2)\} + \varepsilon_\lambda \tag{8}$$

where $I_x(\mathrm{w}_\lambda)$ is the periodogram at the frequencies $\omega_\lambda = 2\pi\lambda/\mathrm{T}(\lambda = 1, \ldots, g(T))$. Under a proper choice of $g(T)$, the ordinary least squares estimator of d is consistent and the distribution of $(\hat{d} - d)/s(\hat{d})$, with $s(\hat{d})$ the standard error $\hat{d}$, is asymptotically normal. The theoretical variance of ε_λ is known to be $\pi^2/6$ and $g(T)$ is commonly selected as $T^{0.5}$.

III. EMPIRICAL RESULTS

Before proceeding any furather, it is informative to consider previous research on stock market behaviour in the Athens Stock Exchange. Papaioannou (1982, 1984) found evidence of dependencies in stock returns over six day intervals. Panas (1990) supports the weak form of the efficient market hypothesis. Panas and Stengos (1992) conducted the Brock, Dechert, and Scheinkman test in order to examine for the peresence of non-linear structure in the residuals of rates of return regressions for a number of selected stocks from the banking sector. In modelling stock returns Koutmos *et al.* (1993) found that an exponential generalized ARCH model proved successful in representing volatility in weekly stock returns.

The data set is based upon daily returns of 13 Greek stocks. In this study the behaviour of stocks will be examined using the continuously compound rates of return, calculated by:

$$Z_{it} = \ln(p_{it}/p_{it-1}) \qquad i = 1, \ldots, 13 \tag{9}$$

where p_{it} is the closing stock price at time t. The data consist of daily stock closing prices, covering the period 4 January 1993 to 5 May 1998; i.e. 1342 trading days.

To test the stationarity of the daily returns, the conventional augmented Dickey–Fuller (1979) τ test is used. The second test used throughout this paper is the Phillips–Perron (1988) test (Z test) which allows for serial correlation and homoscedasticity.

Table 1 contains the values of the augmented Dickey–Fuller and Phillips–Perron statistics for unit root in the log price relatives. Thus, the null hypothesis is that the log price relatives follow a non-stationary process, and the alternative hypothesis is that the log price relatives follow a stationary process. Using the unit root tests advocated by Dickey–Fuller, Phillips–Perron will reject a unit root for all of the stocks and we can conclude that the stocks are stationary. Autocorrelation tests were conducted to determine the linear dependency between log returns at time t and log returns at time $t + k$, for $k = 1, 2, \ldots, 335$. A complete listing of all partial autocorrelations (PACF) and autocorrelations (ACF) is available from the author upon request. The number of each stock which has autocorrelation coefficients significantly different from zero at the 5% level and the highest autocorrelation coefficient are shown in Table 2.

The reported magnitude of autocorrelations and partial autocorrelations ranges between 0.113 and 0.222 which are small, indicating that short memory of the data is weak. Particular attention is focused on autocorrelation coefficients in the neighbourhood of $k = 150, 151, \ldots, 335$. Here, a number of significant partial autocorrelations and autocorrelations at later time lags were found. This indicates that there is dependence among distant observations. More importantly, the long lasting autocorrelations evidence indicates according to Taylor (1986), that the processes are nonlinear with time-varying variances.

In addition, examination of the autocorrelations and partial autocorrelations demonstrates that their patterns do not exhibit significant seasonal fluctuations or definite cycles. The Ljung–Box, Q statistic – see Table 3 – for the log returns series indicates there there is an overall significant autocorrelation in only nine of the thirteen cases. The Ljung–Box Q statistics of the squared return series are statistically significant, except in two cases, indicating that the conditional distributions of the daily returns are changing through time – see Hsieh (1988). This is a symptom of ARCH effects.

To detect the presence of ARCH effects in the sample, Engle's (1982) Lagrange Multiplier (LM) test is used. Engle's proposed LM test can be carried out using the following regression equation:

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + e_t \text{ with } u_t = z_t - \hat{z}_t$$

$$\text{where } \hat{z}_t = \hat{\beta}_0 + \hat{\beta}_1 z_{t-1}$$

The test statistic nR^2, where R^2 from the above regression, is asymptotically distributed as X_1^2 under the $\mathrm{H}_0 : Z_t$ carries no ARCH. Table 3 reports the results of the LM test. The results of this test suggest the presence of ARCH(1) effects in all stock returns and coincide with those of Q-statistic for squared returns. Thus, the Q-statistic for squared returns and the LM test suggest the ARCH specification as a good approximation to the structure of conditional variance of the stock data.

Univariate statistics for daily returns are shown in Table 4.

Skewness is used to assess the symmetry of the distribution, the kurtosis for peakedness, and the fatness of the tails. If the skewness is positive, the distribution is skewed to the right, and if it is negative, the distribution is skewed to the left. The results for the skewness test, reported in Table 4, show that the distributions of daily log price relatives are positive, except in three cases.

As evidenced in Table 4, all log price changes are leptokurtic, i.e., the excess kurtosis is positive. These results support Pagan's (1996) study, in which he argues that the returns of most financial assets have semi-fat tails or, in

Table 1. *The augmented Dickey–Fuller* (τ) *and Phillips–Perron* (Z) *tests*

Stock	τ	Z
Pisteos	−30.3	−109.8
Ktimatiki	−32.6	−120.5
Ellados	−34.6	−126.6
Geniki	−38.6	−141.3
Emporiki	−31.4	−114.4
Ergasias	−30.6	−112.2
Ethniki	−29.3	−104.4
Eteba	−31.5	−114.6
Ioniki	−45.4	−161.4
Iraklis	−34.9	−127.9
Titan	−32.9	−121.0
Alcatel	−32.1	−116.6
Mihaniki	−29.5	−106.1

Table 2. *Structure of daily autocorrelations*

Stock	No of significant ACF	Largest absolute ACF	No of significant PACF	Largest absolute PACF
Pisteos	40	0.197 (1)	29	0.197 (1)
Geniki	17	0.137 (35)	14	0.113 (35)
Ktimatiki	25	0.123 (1)	21	0.123 (1)
Ellados	12	0.113 (3)	11	0.113 (3)
Emporiki	23	0.151 (1)	20	0.151 (1)
Ergasias	36	0.177 (1)	19	0.122 (1)
Ethniki	26	0.222 (1)	17	0.222 (1)
Eteba	15	0.158 (1)	13	0.158 (1)
Ioniki	12	0.217 (1)	18	0.217 (1)
Iraklis	18	0.184 (2)	16	0.181 (2)
Titan	32	0.192 (1)	40	0.192
Alcatel	32	0.136 (1)	24	0.136 (1)
Mihaniki	29	0.211 (1)	21	0.211 (1)

Table 3. *Ljung–Box and Lagrange multiplier statistics*

Stock	Ljung–Box (Returns)	Ljung–Box (Squared Returns)	LM-ARCH
Pisteos	489.9*	1463.2*	126.1*
Geniki	397.2*	667.2*	132.6*
Ktimatiki	388.2*	969.5*	78.2*
Ellados	324.8	358.7	22.94*
Emporiki	440.5*	756.1*	63.0*
Ergasias	461.8*	1135.6*	120.0*
Ethniki	449.5*	1953.7*	109.2*
Eteba	335.6	649.1*	56.2*
Ioniki	321.4	366.5	235.3*
Iraklis	398.6*	670.7*	95.9*
Titan	433.9*	1488.6*	86.2*
Alcatel	341.9	1005.1*	135.0*
Mihaniki	434.3*	607.9*	28.0*

Note: Asterisks denote rejection of the null hypothesis.

Table 4. *Summary statistics, estimates of Levy index and Hurst exponent*

Stock	Summary statistics: Skewness	Excess kurtosis	Bera-Jarque	Levy-index $\hat{\alpha}$	Hurst exponent $\hat{H}$
Pisteos	−0.110	4.753*	1267.8*	1.44*(0.05)	0.694
Geniki	−2.59*	88.24*	43 754*	1.13*(0.07)	0.885
Ktimatiki	0.138*	3.355*	634.3*	1.78(0.09)	0.562
Ellados	−0.657*	10.43*	6191.5*	1.82(0.16)	0.549
Emporiki	0.087	2.981*	499.3*	1.39*(0.034)	0.719
Ergasias	0.073	3.255*	594.5*	1.4*(0.039)	0.714
Ethniki	0.456*	4.733*	1301.1*	1.45*(0.067)	0.690
Eteba	0.087	1.59*	143.3*	1.32*(0.044)	0.758
Ioniki	0.954*	1.66*	359.2*	1.37*(0.055)	0.730
Iraklis	0.219*	1.272*	101.61*	1.41*(0.093)	0.709
Titan	0.333*	3.893*	873.7*	1.34*(0.084)	0.746
Alcatel	0.113	0.746*	34.1*	1.21*(0.019)	0.826
Mihaniki	0.006	2.771*	430*	1.25*(0.022)	0.80

Note: Asterisks denote rejection of the null hypothesis. Standard errors (bootstrap estimates) are in parentheses.

other words, that the actual kurtosis is higher than the zero kurtosis of the normal distribution.

Next, the analysis of the distribution of log price relatives are turned to. The Bera–Jarque (BJ) statistic introduced by Bera and Jarque (1980) can be employed to investigate the normality of log price relatives. The BJ statistic is asymptotically distributed as x^2 under the null hypothesis. If the value of BJ test statistic is greater than the significance point of X_2^2, then the null hypothesis of normality is rejected. The results of the BJ test are presented in Table 4. It may be concluded from Table 4 that the hypothesis that all 13 stocks, empirical distributions of log price relatives are drawn from an underlying normal distribution, is rejected.

The statistical results of Table 4 are consistent with results obtained for daily returns in other studies, for example Koutmos *et al.* (1993), Panas (1990).

In general, from the results of Table 4 the following conclusion is drawn. The findings of leptokurtosis and the significant deviation of the log price relatives from normality can be a symptom of nonlinear dynamics (e.g. Fang *et al.*, 1994). Once normality is rejected, it is appropriate, in this case, to use Levy-stable distributions.

Table 4 summarizes the estimates of Levy-index α. Except for two cases (Ktimatiki and Ellados) it is obvious that all Levy-index αs are different from two, which clearly indicates that normality is not appropriate in describing log price relatives. The values of α range from 1.13 to 1.82, with 11 of the 13 being between 1.13 and 1.45.

Using the bootstrap standard errors of the α-estimates, it is found that two of the thirteen α_s are not significantly different from two. The Levy-index, α has an interesting relationship with Hurst exponent – see Equation 4.

Using Equation 4, the values of the Hurst exponent ($= \hat{H}$) of the stock market time series are presented in Table 4; it ranges from 0.549 to 0.885. In all cases, the values of $\hat{H}$ were greater than 0.5, which implies that the market is a persistent time series, and, therefore, exhibits the Joseph and Noah effects – see Mandelbrot (1972).

More importantly, the results confirm the presence of the long-memory effect since the estimates of the Hurst exponent, $\hat{H}$, are greater than 0.5.

The indirect evidence of the presence of long memory was based on the Hurst exponent, by using the estimated characteristic, α, of the Levy-stable probability density function.

In this study, an alternative technique is used to examine the presence of long memory behaviour of the log price changes that does not depend on knowledge of the underlying distribution. This study, in addition, provides a direct test for fractal dynamics, by using a fractional time series model.

The spectral regression type estimates of the fractional differencing parameter – see Equation 8 – are reported in Table 5.

Fractional differencing estimates are reported for $v = T^{0.55}$ and $v = T^{0.60}$ to evaluate the sensitivity of d esti-

Table 5. *Estimates of the fractional differencing parameter d*

Stock	$\hat{d}$ (0.55)	$\hat{d}$ (0.60)
Pisteos	0.2104[a] (2.4)	0.233[a] (1.8)
Geniki	0.1559[a] (1.86)	0.1706[a] (2.5)
Ktimatiki	0.1534 [a] (2.02)	0.259[a] (3.4)
Emporikh	0.198[b] (1.55)	0.209[a] (1.72)
Ellados	0.186[a] (1.76)	0.151[a] (1.8)
Ergasias	0.1377[a] (2.3)	0.101[a] (2.1)
Ethniki	0.128[a] (2.0)	0.155[a] (2.1)
Eteba	0.103[a] (2.2)	0.111[a] (2.7)
Ioniki	0.152[a] (2.8)	0.168[a] (3.5)
Iraklis	0.2108[a] (2.9)	0.162[a] (2.3)
Titan	0.109[b] (1.58)	0.085 (0.8)
Alcatel	0.121[a] (2.9)	0.104[a] (2.2)
Mihaniki	0.142[a] (2.4)	0.1012[a] (2.3)

Notes: $d(0.55)$ and $d(0.60)$ give the d estimates from spectral regression with sample size $T^{0.55}$ and $T^{0.60}$ respectively. Asterisks denote rejection of the null hypothesis. The values in parentheses are t-values. a = significant at the 5% level ; b = significant at the 10% level.

mates to the choice of the sample size of the spectral regression – see Geweke and Porter-Hudak (1983). The values of $\hat{d}$ range from 0.085 to 0.259.

The statistical results suggest the presence of long memory. Or, in other words, log price change dynamics are well described by a long memory fractional process. Therefore, the spectral regression results constitute, once more, evidence of long memory characteristics of Athens stock returns. Thus, both the indirect and spectral regression analyses provide strong evidence in favour of long memory. Similar evidence of long memory is also found in Barkoulas *et al.* (1998) on weekly stock returns of the Athens Stock Exchange.

The relationship between the fitted ARCH and fractal process is an issue of some practical importance. The empirical results of this section suggest that the Athens Stock Exchange can be appropriately modelled by ARCH and fractal processes. A connection between ARCH and fractal processes has been suggested by Peters (1994): 'ARCH processes are not long memory processes but local processes. Fractal processes, on the other hand, are global processes ... it is possible that the two processes can coexist'.

IV. CONCLUSIONS

This study found, (1) slowly decaying autocorrelation, (2) the presence of heteroscedasticity and, (3) that the distributions of return series are non-normal. These findings provide evidence that the dynamic process generating the daily return series is nonlinear. However, an ARCH process accounts for some of the non-linearities. The Athens Stock Exchange can be characterized, in addition, by other forms of nonlinear dynamics.

The non-normality of distributions leads to the question of the appropriate distribution. The study has tried to demonstrate the utility of stable distributions as applied to investigating fractal patterns in daily returns. This kind of analysis can be valuable in examining a long memory process through the Hurst coefficient, which, once the Levy index is estimated, is fully computed. Given the estimates of the Levy index, the Hurst exponent was evaluated which provides evidence of long memory behaviour in the Athens Stock Exchange.

This paper employed a relatively new approach to the analysis of financial time series. The fractal dynamics allows for interpretation of irregular cyclical fluctuations and long-term dependence. A semi-non-parametric spectral method was adopted in the analysis, in an attempt to estimate the fractional model. The results indicate that there is statistically significant evidence that the return series are described by a long-memory fractional process. Both Levy index and the spectral method found evidence of long memory in stock returns. Therefore, the results of this test provide further evidence of nonlinear characteristics in the sample stock returns.

These findings, thus, reinforce the argument of J. Barkoulas and N. Travlos's (1998) argument that 'the behaviour of Greek stock returns may be consistent with a nonlinear stochastic process...'.

REFERENCES

Aydogan, K. and Booth, G. G. (1988) Are there long cycles in common stock returns? *Southern Economic Journal*, **55**, 141–49.

Barkoulas, J. and Baum, C. F. (1996) Long term dependence in stock returns, *Economics Letters*, **53**, 253–59.

Barkoulas, J. and Travlos, N. (1998) Chaos in an emerging capital market? The case of the Athens Stock Exchange, *Applied Financial Economics*, **8**, 231–41.

Bera, A. K. and Jarque, C. M. (1980) Efficient test for normality, homoscedasticity and serial independence of regression residuals, *Economic Letters*, **6**, 255–9.

Beran, J. (1994) Statistics for Long-Memory Processes, Chapman and Hall, New York.

Blattberg, R. C. and Gonedes, N. J. (1974) A comparison of the stable and student distributions as statistical models of stock prices, *Journal of Business*, **47**, 244–80.

Box, G. E. P. and Jenkins, G. M. (1976) Time Series Analysis Forecasting and Control, San Francisco, Holden-Day.

Brockwell, P. J. and Davis, R. A. (1991) *Time Series: Theory and Methods*, Springer-Verlag, New York.

Cheung, Y. W. and Lai, K. S. (1993) Do gold market returns leave long memory? *The Financial Review*, **28**, 181–202.

Cheung, Y. W. and Lai, K. S. (1995) A search for long memory in international stock market returns, *Journal of International Money and Finance*, **14**, 597–615.

Dickey, D. A and Fuller, W. A. (1979) Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association*, **74**, 427–31.

Efron, B. (1979) Bootstrap Methods: Another Look at the Jackknife, *Annals of Statistics*, **7**, 1–26.

Fama, E. F. (1965) The behaviour of stock market prices, *Journal of Business*, **38**, 34–105.

Fama, E. F. and Roll, R. (1968) Some properties of symmetric stable distributions, *Journal of the American Statistical Association*, **63**, 817–836.

Fama, E. F. and Roll, R. (1971) Parameter estimates for symmetric stable distributions, *Journal of the American Statistical Association*, **66**, 331–338.

Fang, H. K., Lai, S. and Lai, M. (1994) Fractal structure in currency futures price dynamics, *The Journal of Futures Markets*, **14**, 169–181.

Feller, W. (1971) An Introduction to Probability Theory and its Applications Vol. II Second Edition, John Wiley, New York.

Fielitz, B. D. and Rozelle, J. (1983) Stable distributions and the mixtures of distributions hypotheses for common stock returns, *Journal of the American Statistical Association*, **78**, 28–36.

Fielitz, B. D. and Smith, E. W. (1972) Asymmetric stable distributions of stock price changes, *Journal of the American Statistical Association*, **67**, 813–14.

Geweke, J. and Porter-Hudak, S. (1983) The estimation and application of long memory time series models, *Journal of Time Series Analysis*, **4**, 221–38.

Granger, C. W. J. and Joyeux, R. (1980) An introduction to long-memory time series models and fractal differencing, *Journal of Time Series Analysis*, **1**, 15–29.

Hosking, J. R. M. (1981) Fractal differencing, *Biometrica*, **68**, 165–176.

Hsieh, D. A. (1988) The stastistical properties of daily foreign exchange rates: 1974–1983, *Journal of International Economics*, **24**, 129–145.

Hsu, D., Miller, R. and Wickern, D. (1974) On the stable paretian behaviour of stock market prices, *Journal of the American Statistical Association*, **69**, 108–13.

Hurst, H. E. (1951) Long-term storage capacity of reservoirs, *Transaction of the American Society of Civil Engineers*, **116**, 770–99.

Jarque, C. M. and Bera, A. K. (1980) Efficient test for normality, homoscedasticity and serial independence of regression residuals, *Economic Letters*, **6**, 255–9.

Kon, S. (1984) Models of stock returns–A Comparison, *Journal of Finance*, **39**, 147–65.

Koutmos, G., Negakis, C. and Theodosiou, P. (1993) Stochastic behaviour of the Athens Stock Exchange, *Applied Financial Economics*, **3**, 119–26.

Levy, P. (1937) Theorie de l' Addition des Variables Aleatoires, Paris: Gauthier-Villars.

Lo, A. W. (1991) Long term memory in stock market prices, *Econometrica*, **59**, 1279–1313.

Lo, A. and Mackinlay, A. C. (1988) Stock market prices do not follow random walks: Evidence from simple specification test, *Review of Financial Studies*, **1**, 41–66.

Mandelbrot, B. (1963) The variation of certain speculative prices, *Journal of Business*, **36**, 394–419.

Mandelbrot, B. (1972) Statistical methodology for non-periodic cycles: From the covariance to R/S analysis, *Annals of Economics and Social Measurement*, **1**, 259–90.

Mandelbrot, B. (1977) Fractals and Scaling in Finance, Berlin: Springer-Verlag.

Pagan, A. (1996). The econometrics of financial markets, *Journal of Empirical Finance*, **3**, 15–102.

Panas E. (1990) The behaviour of the Athens Stock Prices, *Applied Economics*, **22**, 1715–27.

Panas, E. and Stengos, T. (1992) Testing the efficiency of the Athens Stock Exchange: Some results from the Banking Sector, *Empircal Economics*, **17**, 239–52.

Papaioannou, G. J. (1982) Thinness and short-run price dependence in the Athens Stock Exchange, *Greek Economic Review*, 315–33.

Papaioannou, G. J. (1984) *Informational Efficiency Tests in the Athens Stock Market.* In G. A. Hawawini and P. A. Michel, (eds), *European Equity Markets: Risk, Return, and Efficiency*, Garland Publishing, New York, pp. 367–81.

Papaioannou, G. J. and Philippatos, G. C. (1982) The intervaling effect under non-synchronous trading and price adjustment lags in the Athens stock exchange, *Spoudai*, **4**, 627–39.

Peters, E. E. (1991) Chaos and Order in the Capital Markets, John Wiley, New York.

Peters, E. E. (1992) R/S Analysis using logarithmic returns, *Financial Analyst Journal*, **48**, 32–37.

Peters, E. E. (1994) Fractal Market Analysis, John Wiley, New York.

Philips, P. C. B. and Perron, P. (1988) Testing unit roots in time series regression, *Biometrica*, **75**, 335–46.

Press, S. (1972) Estimation in univariate and multivariate stable distributions, *Journal of the American Statistical Association*, **67**, 842–6.

Simkowitz, M. A. and Beedles, W. L. (1980) Asymmetric stable distributed security returns, *Journal of the American Statistical Association*, **75**, 306–12.

Taylor, S. (1986) Modelling Financial Time Series, John Wiley, New York.

Theodosiou, P., Koutmos, G. and Negakis, C. (1993) The intertemporal relation between the US and Greek stock markets: A conditional tale analysis, *The International Journal of Finance*, **6**, 492–508.

Zolotarev, V. (1986) One Dimensional Stable Distributions, Providence, R. I. American Mathematical Society.