R/S Analysis and Long Term Dependence in Stock Market Indices

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ABSTRACT

Recent studies indicating long term dependence in stock market indices have found a mean reversion process. However, studies using rescaled range (R/S) analysis have not found evidence of a mean reversion or ergodic process. Instead, evidence from these studies indicate either long term persistence in a nonperiodic cycle or short run Markovian dependence with no long term persistence. The purpose of this paper is to study the issue of long term dependence using rescaled range analysis. The empirical results obtained in this study support the persistent dependence/nonperiodic cycle results and suggest that the dependence arises from the general economic cycle.

INTRODUCTION

With the publication of the Peters(1991) text on chaos theory and the stock market, there has been substantial interest and controversy in the area of long term market dependence. Using rescaled range-Hurst regression analysis, Peters(1991) found long term persistent dependence with finite nonperiodic cycles in stock market indices instead of the mean reversion process found in other stock market studies. Additionally, several studies using the Lo(1991) modified rescaled range (R/S) test have contradicted Peters'(1991) results. The purpose of this paper is to identify the issues of contention and to use Peters'(1991) R/S Hurst analysis and Lo's(1991) modified R/S test to study long term dependence in stock prices.

The paper is organized as follows. The first section discusses the issues involved in using rescaled range analysis. Section 2 presents the methodology used in this paper followed by the empirical results in the third section. The last section contains concluding remarks.

I. REVIEW OF THE ISSUES

Dependence over nonperiodic cycles is defined as the presence of extended periods of similar behavior which are of unequal duration [Booth, Kaen and Koveos(1982)]. Mandelbrot(1972) argues that rescaled range (R/S) analysis can detect nonperiodic cycles even when the cycles have lengths greater than or equal to the sample period. The importance of Mandelbrot's(1972) argument is that it raises the question of whether R/S analysis can be used to detect long term dependence in stock prices.

Long term dependence to Mandelbrot(1972) means the "Joseph effect", named after the Old Testament prophet who foretold of seven years of prosperity followed by seven years of famine [Mandelbrot and Wallis(1968)]. The "Joseph effect" implies that a time series has infinite memory, that is, an event occurring today will still have an effect on events occurring into perpetuity. In studies of geophysical records, Mandelbrot and Wallis(1969) found a number of series with infinite memory. However, the type of time series found in this field very possibly has finite memory cycles that are longer than their time samples, and hence, the infinite memory result.

Mandelbrot(1971) was the first to suggest that R/S analysis could be useful in studies of economic data and provided an economic rationale. In Mandelbrot(1972), it was further argued that R/S analysis was superior to autocorrelation and variance analysis since it could consider distributions with infinite variance and was superior to spectral analysis because it could detect nonperiodic cycles.

The problem with Mandelbrot's analysis was the adherence to processes with infinite memory. In the mathematics of fractal geometry developed in Mandelbrot(1982), fractals will continue to scale to infinity. Peters(1991, p.82), on the other hand, argues that in nature fractals will stop scaling at a finite point (e.g. the passageways in your lungs will stop branching at some finite point). Consistent with Peters(1991), it can be reasonably argued that economic time series have finite memory and R/S analysis must be used over subperiods in order to discover the length of the finite memory or the average nonperiodic cycle. Most academic studies to this point have assumed Mandelbrot's infinite memory process and perform the R/S analysis only on the complete sample.

Mandelbrot, however, does acknowledge the existence of finite memory. In Mandelbrot and Wallis(1969), it is noted that observations far removed in time can be considered independent and that the R/S analysis will asymptotically approach a random process. With shorter lags, the dependence will be evident, but a "break" will occur at longer lags and independence will be obtained. Since Mandelbrot and Wallis(1969) do not observe such a "break" in geophysical records, they consider, for practical purposes, that these time series exhibit infinite memory. Mandelbrot(1972) discusses that there can be short term R/S dependence where a time series has a finite but long memory. It may well be that the time series has a finite memory and R/S analysis will indicate dependence, but, at longer lags, a "break" toward random behavior occurs. From a very long run viewpoint, Mandelbrot(1972) considers this dependence to be a special transient, but goes on to say that this does not lessen the importance of the finite memory component. In fact, Mandelbrot and Wallis(1969), as well as Peters(1991), use R/S analysis to detect the well known 11 year cycle in sunspot activity. They add a warning that processes with a strong periodic element will affect the Hurst phenomenon, but again they are examining the data for infinite memory and feel that these "subharmonics" complicate the issue. In economics, following Peters'(1991) argument we would expect to find finite memory processes, and the "break" in the R/S analysis detects these finite memory nonperiodic cycles.

Peters(1991) uses R/S analysis and a Hurst(1951) regression to examine stock market indices for persistent finite memory and finds evidence of a four year cycle. However, his analysis may be biased by short term Markovian dependence. Davies and Harte(1987) show that conventional R/S analysis using a Hurst regression can be biased toward accepting a long term dependence hypothesis even when the true process is first order autoregressive. As a result, Lo(1991) developed a modified R/S test that allows for short-term dependence, nonnormal distributions, and conditional heteroscedasticity under the null hypothesis. In addition, Cheung(1993) uses Monte Carlo simulation to show that the modified R/S test is robust to nonstationary variance and ARCH (autoregressive conditional heteroscedasticity) effects. The only problem is that the Lo(1991) modification does assume an infinite memory process. Fortunately, like R/S analysis, it too can be used on different subperiods [Cheung and Lai(1993)].

II. METHODOLOGY

The rescaled range was developed by Hurst(1951). The R/S statistic is the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. Given a sample of returns, X1, X2,. . . Xn, for n periods and a sample mean Xn, the classical rescaled range is:

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$$R/S_n = \frac{1}{S_n}\left| \max_{1<k<n} \sum_{j=1}^{k} (X_j - \bar{X}_n) - \min_{1<k<n} \sum_{j=1}^{k} (X_j - \bar{X}_n) \right| \quad (1)$$

where Sn is the standard deviation estimator. Hurst found that the observations appeared to be well represented by the relation:

$$R/S_n = a n^H \quad (2)$$

where H is the Hurst exponent. Using a logarithmic transformation, the Hurst exponent can be estimated using the following regression:

$$\text{Log}(R/S_n) = \text{Log}(a) + H(\text{Log}(n)) \quad (3)$$

Through Monte Carlo simulation, Hurst noted that if the underlying process is a random draw from a stable distribution, then H = 0.5. If H is greater than 0.5, there is evidence of persistent dependence (large values followed by large values and small values followed by small values) and if H is less than 0.5, an ergodic or mean reverting process is indicated. The infinite memory result implies that H will stabilize asymptotic to some value other than 0.5 and will maintain that value no matter how large the sample size. Peters(1991), on the other hand, states that in a finite memory process, H will stabilize on a value other than 0.5 within a finite sample size and then will "break" and move asymptotically toward a value of 0.5 as the sample size increases.

The problem with estimating the Hurst exponent with regression analysis is that the regression coefficients may be biased as a result of autocorrelation. Furthermore, the traditional R/S value is not acceptable since tests of the statistical significance do not exist [Lo(1991)]. Therefore, Lo(1991) developed a modified R/S statistic where Sn is adjusted for short term dependence:

$$S_n(q) = \frac{1}{n}\sum_{j=1}^{n}(X_j - \bar{X}_n)^2 + \frac{2}{n}\sum_{j=1}^{q} w_j(q)\left[\sum_{i=j+1}^{n}(X_i - \bar{X}_n)(X_{i-j} - \bar{X}_n)\right]^{0.5} \qquad (4)$$

where

$$j(q) = 1 - \left|\frac{j}{(q+1)}\right|, \quad q<n \qquad (5)$$

and where q is the number of lags in the weighted autocovariance function used to adjust Sn. Instead of computing R/S statistic as R/Sn, Lo computes the modified R/S statistic as R/Sn(q). Both statistics are normalized by the number of observations and the test statistics Vn and Vn(q) may be presented as:

$$V_n = \frac{1}{\sqrt{n}}\,\frac{R_n}{S_n} \qquad (6) \qquad\qquad V_n(q) = \frac{1}{\sqrt{n}}\,\frac{R_n}{S_n(q)} \qquad (7)$$

Vn is the traditional R/S statistic and is equivalent to the Lo(1991) modified R/S statistic where q is equal to zero. These test statistics can be evaluated for significance using the critical values from Lo(1991) given below in Table 1.

Table 1 - Fractiles of the Distribution

Prob.	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	0.95	0.975	0.995
Vn	0.927	1.018	1.090	1.157	1.223	1.294	1.374	1.473	1.620	1.747	1.862	2.098

Vn and Vn(q) are computed for each sample size n and therefore, are not dependent on regression analysis in order to be estimated. If the process being investigated has an infinite memory, both Vn and Vn(q) will converge asymptotically to stable values which can be evaluated for significance. If the process has a finite memory, the Lo(1991) statistics should reach a maximum value at a finite sample size and then converge asymptotically to a value that indicates a random walk.

In performing the empirical analysis, subsamples of various sizes are examined. These subsamples can be independent (nonoverlapping subsamples), corresponding to the original Hurst approach, or they can be overlapping (or nested) samples in order to maximize the number of observations. If the samples are overlapping, the F-Hurst and G-Hurst methods, as suggested in Wallis and Matalas(1970), can be employed to test for dependence. Using independent subsamples, Mandelbrot and Wallis(1969) suggest that the smaller subsample sizes be ignored because of initial transient behavior (Markovian dependence). Larger sample sizes can also be ignored since the two choices available in constructing these subsamples, very few nonoverlapping subsamples or a larger number of overlapping subsamples with their resulting intercorrelations, are both unacceptable.

The size of the sample is important. Aydogan and Booth(1988) suggest that there should be 15 subsamples for each sample size n. Peters(1991, p.114) counters that the number of observations for each sample size is important only if the process being studied is independently and identically distributed (iid). Lo(1991) notes that there is a consensus in the financial economics literature that stock market prices are not iid. Importantly, Peters(1991) states that the time series should be long enough to contain at least 10 potential cycles or finite memory periods. On the other hand, having a huge number of observations is much less important. In other words, time matters, not observations.

This study uses the CRSP monthly value-weighted index from January 1926 to December 1992 (804 monthly observations for 67 years) and the S&P 500 daily index from July 1962 to December 1991 (7420 daily observations for 29.5 years). Since Peters(1991) detected 4 year cycles, the monthly data set provides a sufficient time period.

The independent subsample approach of Peters(1991) is also used since it is closer to the original Hurst-R/S analysis and it reduces potential correlations between subsamples. Additionally, Anis and Lloyd(1975) found that independent summands used in calculating rescaled range are more robust than correlated summands. In the empirical analysis, sample size n starts at a value large enough to minimize short term Markovian dependence and stops when only one subsample for n is available. (Therefore, the largest value of n will be one-half of the available number of observations.) The regression used to estimate the Hurst exponent is performed and the Lo(1991) Vn statistic is calculated for each individual sample size n. The results are examined for local maxima for both the Hurst and the Lo statistics. This is consistent with Peters'(1991) argument that economic processes exhibit finite memory.

III. Empirical Results

Table 2 presents the Hurst and Lo statistics using monthly CRSP value-weighted index returns from January 1926 to December 1992. The smallest sample size tested is 24 months. This represents a tradeoff. Numerous authors [Booth, Kaen and Koveos(1982), Aydogan and Booth(1988), and Ambrose, Ancel and Griffiths(1993)] recommend that the Hurst method start with a larger sample size. Too small a sample allows short term transients (Markovian dependence) to affect the results [Mandelbrot and Wallis(1969)], but too long a starting sample period would not permit the detection of Peters'(1991) four year (48 month) cycle. A 24 month cycle is long enough to ignore short term cycles (most studies mentioned above use daily data and a starting sample size of 50 days), but short enough to allow for sufficient degrees of freedom.

Place Table 2 Here

Table 2 focuses on sample sizes of 48 months because of Peters'(1991) results, 64 months because this is where the Hurst statistic is maximized, and 114 months as a result of the daily findings to be presented in Table 3. For readers interested only in infinite memory processes, the complete sample size of 402 months is also presented.

The evidence suggests that there is a local maximum in the Hurst exponent at around 50 months. This is confirmed by the t-statistic, which achieves a local maximum at 50 months, a Durbin-Watson statistic which indicates no autocorrelation in the regression, and significant Lo statistics for lags of 0, 1, 3, 6 and 12 months.

There are global maxima for the Hurst exponent and the t-statistic for sample sizes of 64-65 months. This is confirmed by significant Lo statistics at 64 months for all lags. The interesting part of this section of Table 2 is that the "break" can be observed after n=64 months. The Durbin-Watson statistic starts to deteriorate and shows significant autocorrelation by n=69 months. The Lo statistics also decline in both magnitude and significance. By 402 months, the Hurst statistic is 0.55, the Durbin-Watson statistic is 0.30, and the Lo statistics reveal no long term dependence. This is the classic case described by Peters(1991).

The problem with the "break" is that the slope of the regression changes after 64 months and the linear relationship between R/S and n before n=64 becomes nonlinear after n=64. Fitting a straight line through nonlinear data is a classic cause of significant autocorrelation. By the time n=402, the Durbin-Watson statistic is 0.30 and it is very doubtful that the Hurst exponent can be considered an unbiased estimator. However, the Lo statistic is adjusted for autocorrelation effects and does not have this problem. It is interesting to note that before n=64, when the Hurst regression is free of one period lag autocorrelation effects, the Lo statistic confirms the Hurst exponent. However, at 114 months, where the next Hurst local maximum occurs, the Lo statistic does not confirm the Hurst statistic. The maximum Lo statistic occurs at 112 months for q equal to 0, 1, and 3. Examining maximum values, the Lo statistic indicates a finite memory cycle of 49-50 months while the Hurst exponent indicates a finite memory cycle of 64 months. There is also evidence of a weaker cycle at around 10 years (120 months).

In Table 3, the effect of short term dependence (autocorrelation) on the Hurst exponent is very evident. Table 3 presents the results of the daily S&P 500 Composite index from July 1962 to December 1991. The smallest sample size is 130 days, which should eliminate any short term transient factors. First, all of the Hurst exponents for the 48, 60 and 120 month cycles detected in Table 2 are associated with regressions possessing severe autocorrelation (Durbin-Watson statistics vary from 0.02 to 0.27). Next, the Hurst exponent and t-statistic is maximized at around 1340 days (5 years) while the Lo statistics is maximized at 2524 days (10 years). Also, the Lo statistics have a local maximum at n=1316, whereas the Hurst is maximized at n=1341 and the t-test is maximized at n=1345.

Place Table 3 Here

The daily data reverses the results from the monthly data. The 4 and 5 year cycles are weaker while the 10 year cycle is stronger. This conclusion comes from the Lo statistic, which is more robust than the Hurst statistic. For the 4 and 5 year cycles, the Lo statistic is not statistically significant when the lag is increased to 5 days. The 10 year cycle maintains statistical significance to q=20 (and also to q=30 which is not presented in the table). Again, there is no evidence of infinite memory with either the Hurst exponent or the Lo statistic when n=3710. Even so, with two different indexes for two different types of data, the 4, 5, and 10 year cycles are evident.

To Mandelbrot and Wallis(1969), these subharmonics complicate the attempt to find infinite memory. To Peters(1991), it doesn't make sense to try to find infinite memory in economic processes since fractals cannot scale to infinity in economics as they can in mathematics. To Peters(1991), these cycles are finite memory nonperiodic cycles and indicate that stocks have long term memory. To others, these cycles are short term transients that must be controlled in order to test for long term memory. For example, Goetzmann(1993), using three centuries of stock market index data, employs a 20 year moving average to detrend the data and then test for long term dependence.

Still, this issue needs to be explored further. Aydogan and Booth(1988) warn that these shorter term cycles (or preasymptotic behavior) represent a nonstationary mean process. In fact, Cheung(1993) tests the Lo statistic using Monte Carlo simulation and finds it is very sensitive to nonstationary means, while at the same time it is robust to nonstationary variance. This result reflects the fact that the modified R/S (Lo) statistic assumes a constant mean but allows for changing variances under the null hypothesis. This finding is interesting since I have argued that financial markets following a bifurcation theory process will be characterized by a mean jump process as well as a nonstationary covariance process (Nawrocki, 1984). This theory is inconsistent with the ARMA, ARIMA, ARCH, GARCH, EGARCH, catastrophe and chaos theory approaches, which assume a stationary mean.

Even so, the question of these subharmonics deserves further investigation. Lo(1991, p.1308) makes a fascinating observation at the end of his article (which dismisses long term persistence in security prices) in stating, "Perhaps the fluctuations of aggregate economic output are more likely to display such long-run tendencies, as Kondratiev and Kuznets have suggested, and this long-memory in output may eventually manifest itself in return to equity." Kondratiev, to refresh memories, found 4, 10 and 50 year cycles in economic output data. Since Tables 2 and 3 indicate 4 and 10 year cycles, is this an interesting coincidence or is there a relationship between general economic cycles and stock market dependence?

Previous researchers such as Groth(1979) and Morse(1980), have found that security dependence varies over time and with the volume of trading. Consider the following: the higher the volume of trading, the more information which arrives in the market. The increase in information causes increased uncertainty and trading at disequilibrium prices. This results in a competence-difficulty (C-D) gap [Heiner(1983) and Kaen and Rosenman(1986)], defined as the spread between the competence of the investors and the complexity of the information. Several important questions related to the C-D gap can be posed. When does the greatest level of uncertainty occur in the market? When do liquidity problems occur in the market, causing anomalies like humped yield curves? The answers to these questions is when economy is entering into a recession or credit crunch. The 4-5 year cycles are thus probably tied to recessions and credit crunches.

In testing this hypothesis, a recent article by Cheung and Lai(1993) provides some insight. Booth, Kaen and Koveos(1982) found long term dependence in gold prices but were concerned about a short term transient that may have been caused by the events in Iran and the Hunt brothers' silver price manipulations in late 1979. Cheung and Lai(1993) use the Lo statistic in a rolling sample approach and find that there is no dependence in gold prices before or after 1979. Simply by eliminating the November and December 1979 observations, significant R/S statistics become insignificant. Cheung and Lai(1991) attribute this result to nonstationary means during the late 1979 period and conclude that the rescaled range is very sensitive to nonstationary means in that only a few observations can result in a hypothesis of significant persistent dependence to be accepted. If so, eliminating data periods associated with declining economic output or declining money supply should result in the disappearance of the 4-5 year cycles found in this study.

The industrial production index and the real M1 money supply variables were obtained from the Citibase databank for the period January 1947 to December 1992. Since the real dollar production index did not start until 1971, the industrial production index was adjusted to real dollar terms using the consumer price index. Monthly periods in which these indexes showed a decline were eliminated from the data set and the R/S analysis was performed on the remaining observations. The results are presented in Table 4. The CRSP monthly index contains 552 monthly observations providing 276 samples. Again, the smallest sample size is n=24.

Place Table 4 Here

The unadjusted CRSP Index shows maximum values for the Hurst exponent and the Lo statistics at around 48-49 months and 63-64 months. All statistics are significant with no autocorrelation, as indicated by the Durbin-Watson test. When the CRSP index is adjusted for negative changes in the nominal industrial production index, the 48-49 month cycle is still present in both the Hurst and Lo statistics but the 64-65 month cycle statistics are not significant (the Hurst exponent is plagued with significant autocorrelation while the Lo statistics are insignificant with the exception of Vn(12)).

However, the inflation adjusted production index is probably more pertinent to this study. When the industrial production index is adjusted using the consumer price index, for the 48-49 month period, all of the Hurst exponents decline toward H=0.50, the t-tests are smaller in magnitude and the Lo statistics decline (only Vn(6) and Vn(12) remain significant.) For the 64-65 month period, the Hurst exponent is actually showing ergodic behavior but this is questionable because of autocorrelation and insignificant Lo statistics. However, the Lo statistics reveal mean reverting (ergodic) behavior for the entire period, the only time that significant ergodic behavior is detected in this paper.

Because sufficient liquidity is important to the clearing of financial markets, a test using the money supply should prove interesting. When periods of declining real money supply are eliminated, the Hurst regressions all exhibit significant autocorrelations and the Lo statistics are insignificant for shorter lags for the 48-49 month samples and for all lags for the 64-65 month samples. The complete period also does not show any significance. An anomaly in the results for real industrial production and real money supply occurs for the 47-48 month cycle, as indicated by the significant Vn(6) and Vn(12) statistics. Otherwise, differences between the unadjusted and unadjusted CRSP index statistics indicate that negative real changes in industrial production and the money supply account for most of the 48 month cycle and all of the 64-65 month cycle.

IV SUMMARY AND CONCLUSIONS

There is no evidence found of persistent long term memory in stock prices given the Mandelbrot infinite memory approach. However, if fractals scale to a finite level in nature and economics, then we would not expect to find this type of dependence. Thus, Peters(1991) would be correct in stating that a finite memory model is more appropriate to studies of the financial markets.

Using daily and monthly market indices, the Hurst exponent and the Lo modified R/S statistic indicate that there is persistent finite memory. Since the nonperiodic cycles do change, the average cycles found are at around 4, 5 and 10 years. The four and five year cycles are shown to be sensitive to negative changes in real industrial production and in the real M1 money supply. This type of behavior is consistent with a nonstationary mean process in the markets and indicates that long term persistent (finite memory) dependence in the general economic cycle does find its way into stock prices. I would add a strong word of caution to future studies in that they should not assume away nonstationary mean processes when studying the financial markets.

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Table 2
R/S Analysis, Hurst Regressions, Lo Vn(q) Statistics for CRSP Value-Weighted Index for January 1926 to December 1992.
(Smallest Sample Size n=24 Months and Largest Sample Size n=402 Months)

n	Hurst	T-Test	D.W.	Vn(0)	Vn(1)	Vn(3)	Vn(6)	Vn(12)
48	.6912	123.15	1.68	1.720(.90)	1.693(.90)	1.707(.90)	1.700(.90)	1.818(.95)
49	.6965	131.39	1.71	1.766(.95)	1.729(.90)	1.745(.90)	L1.755(.95)	L1.869(.975)
50	.7047L	139.72L	1.64	L1.782(.95)	L1.762(.95)	L1.765(.95)	1.736(.90)	1.722(.90)
51	.6956	134.98	1.75	1.667(.90)	1.660(.90)	1.672(.90)	1.683(.90)	1.794(.95)
52	.6936	138.98	1.77	1.698(.90)	1.676(.90)	1.682(.90)	1.676(.90)	1.775(.95)
63	.7157	202.20	1.66	1.671(.90)	1.632(.90)	1.638(.90)	1.613(.80)	1.653(.90)
64	.7209L	206.52	1.65	L1.750(.95)	L1.705(.90)	L1.713(.90)	L1.674(.90)	L1.707(.90)
65	.7188	209.41G	1.75	1.658(.90)	1.615(.80)	1.619(.80)	1.604(.80)	1.663(.90)
66	.7155	208.78	1.71	1.635(.90)	1.591(.80)	1.591(.80)	1.575(.80)	1.596(.80)
67	.7063	185.23	1.48	1.550(.80)	1.518(.80)	1.527(.80)	1.518(.80)	1.569(.80)
68	.7019	181.91	1.44*	1.601(.80)	1.580(.80)	1.601(.80)	1.594(.80)	1.623(.90)
69	.6977	178.94	1.39+	1.596(.80)	1.561(.80)	1.580(.80)	1.561(.80)	1.587(.80)
111	.6216	125.46	0.73+	1.583(.80)	1.545(.80)	1.540(.80)	1.513(.80)	1.495(.80)
112	.6232	127.80	0.72+	L1.611(.80)	L1.571(.80)	L1.568(.80)	1.536(.80)	1.526(.80)
113	.6243	130.11	0.72+	1.585(.80)	1.546(.80)	1.547(.80)	1.525(.80)	1.537(.80)
114	.6254L	132.48L	0.71+	1.595(.80)	1.559).80)	1.563(.80)	L1.538(.80)	L1.542(.80)
115	.6236	130.85	0.75+	1.465(.70)	1.431(.70)	1.429(.70)	1.417(.70)	1.445(.70)
116	.6219	129.38	0.74+	1.465(.70)	1.427(.70)	1.420(.70)	1.392(.70)	1.389(.70)
117	.6195	126.01	0.71+	1.433(.70)	1.397(.70)	1.388(.70)	1.364(.60)	1.462(.60)
118	.6191	127.13	0.74+	1.520(.80)	1.474(.80)	1.457(.70)	1.415(.70)	1.374(.70)
119	.6175	125.89	0.74+	1.462(.70)	1.431(.70)	1.427(.70)	1.399(.70)	1.383(.70)
120	.6160	124.77	0.73+	1.461(.70)	1.432(.70)	1.429(.70)	1.392(.70)	1.378(.70)
402	.5500	112.38	0.30+	1.366(.60)	1.310(.60)	1.320(.60)	1.311(.60)	1.279(.50)

G - Global Maximum
L - Local Maxima
+ - Significant positive autocorrelation (Durbin-Watson Test)
* - Indeterminate Durbin-Watson Test
Probability in parenthesis is Lo test from Table 1

Table 3
R/S Analysis, Hurst Regressions, Lo Vn(q) Statistics for S&P 500
Composite Index for July 1962 to December 1991
(Smallest Sample Size n=130 Days and Largest Sample Size n=3710 Days)

n	Hurst	T-Test	D.W.	Vn(0)	Vn(1)	Vn(5)	Vn(10)	Vn(20)
1063	.6122	773.50	0.27+	1.763(.95)	1.625(.90)	1.547(.80)	1.537(.80)	1.541(.80)
1064	.6123	773.10	0.27+	1.785(.95)	1.645(.90)	1.568(.80)	1.558(.80)	1.552(.80)
1065	.6125	771.50	0.27+	1.809(.95)	1.666(.90)	1.590(.80)	1.584(.80)	1.581(.80)
1066	.6128	770.38	0.27+	L1.828(.95)	L1.686(.90)	L1.610(.80)	L1.603(.80)	1.599(.80)
1067	.6130	769.07	0.26+	1.826(.95)	1.683(.90)	1.608(.80)	1.602(.80)	L1.600(.80)
1068	.6132	768.22	0.26+	1.791(.95)	1.651(.90)	1.573(.80)	1.567(.80)	1.566(.80)
1069	.6134	767.87	0.26+	1.805(.95)	1.664(.90)	1.587(.80)	1.581(.80)	1.580(.80)
1313	.6209	941.15	0.22+	1.801(.95)	1.660(.90)	1.584(.80)	1.580(.80)	1.586(.80)
1314	.6210	941.03	0.22+	1.816(.95)	1.674(.90)	1.599(.80)	1.545(.80)	1.601(.80)
1315	.6211	941.53	0.22+	1.800(.95)	1.660(.90)	1.583(.80)	1.579(.80)	1.586(.80)
1316	.6212	941.11	0.22+	L1.818(.95)	L1.677(.90)	L1.603(.80)	L1.599(.80)	L1.606(.80)
1317	.6213	940.42	0.22+	1.817(.95)	1.676(.90)	1.602(.80)	1.598(.80)	1.605(.80)
1318	.6214	940.60	0.21+	1.806(.95)	1.666(.90)	1.592(.80)	1.588(.80)	1.595(.80)
1319	.6215	940.50	0.21+	1.775(.95)	1.638(.90)	1.565(.80)	1.560(.80)	1.568(.80)
1340	.6217	961.49	0.22+	1.671(.90)	1.541(.80)	1.471(.70)	1.466(.70)	1.475(.70)
1341	.6217G	961.73	0.22+	L1.672(.90)	L1.543(.80)	L1.473(.70)	L1.468(.70)	L1.477(.70)
1342	.6216	962.12	0.21+	1.662(.90)	1.534(.80)	1.466(.70)	1.460(.70)	1.469(.70)
1343	.6216	962.07	0.21+	1.655(.90)	1.528(.80)	1.460(.70)	1.454(.70)	1.464(.70)
1344	.6215	961.91	0.21+	1.652(.90)	1.526(.80)	1.458(.70)	1.453(.70)	1.462(.70)
1345	.6215	962.15G	0.21+	1.660(.90)	1.532(.80)	1.464(.70)	1.459(.70)	1.477(.70)
1346	.6214	961.94	0.21+	1.642(.90)	1.515(.80)	1.447(.70)	1.442(.70)	1.449(.70)
1347	.6213	961.87	0.21+	1.633(.90)	1.506(.80)	1.438(.70)	1.434(.70)	1.440(.70)
1348	.6212	960.02	0.21+	1.618(.80)	1.491(.80)	1.423(.70)	1.417(.70)	1.421(.70)
2521	.5707	366.58	0.03+	1.921(.98)	1.743(.90)	1.631(.90)	1.624(.90)	1.612(.80)
2522	.5708	366.91	0.03+	1.940(.98)	1.759(.95)	1.647(.90)	1.640(.90)	1.627(.90)
2523	.5709	367.27	0.03+	1.955(.98)	1.773(.95)	1.659(.90)	1.652(.90)	1.638(.90)
2524	.5711	367.62	0.03+	G1.970(.98)	G1.787(.95)	G1.672(.90)	G1.663(.90)	G1.649(.90)
2525	.5712	367.79	0.03+	1.969(.98)	1.786(.95)	1.670(.90)	1.662(.90)	1.649(.90)
2526	.5713	368.00	0.03+	1.923(.98)	1.745(.90)	1.630(.90)	1.624(.90)	1.615(.80)
2527	.5714	368.28	0.03+	1.917(.98)	1.739(.90)	1.625(.90)	1.618(.80)	1.610(.80)
3710	.5371	200.36	0.02+	1.423(.70)	1.365(.60)	1.277(.50)	1.283(.50)	1.279(.50)

G - Global Maximum
L - Local Maxima
+ - Significant positive autocorrelation (Durbin-Watson Test)
* - Indeterminate Durbin-Watson Test
Probability in parenthesis is Lo test from Table 1

Table 4

R/S Analysis, Hurst Regressions, Lo Vn(q) Statistics for CRSP Value-Weighted Index for January 1947 to December 1992

(Smallest Sample Size n=24 Months and Largest Sample Size n=276 Months

Unadjusted CRSP Inde

n	Hurst	T-Test	D.W.	Vn(0)	Vn(1)	Vn(3)	Vn(6)	Vn(12)
48	.7711G	238.84	2.79*	1.725(.90)	1.728(.90)	1.765(.95)	1.722(.90)	1.763(.95)
49	.7691	247.17	2.78*	G1.731(.90)	G1.732(.90)	G1.751(.95)	L1.723(.90)	L1.860(.95)
50	.7622	243.16	2.64	1.690(.90)	1.688(.90)	1.719(.90)	1.697(.90)	1.820(.95)
63	.7356	274.17	2.25	L1.688(.90)	1.660(.90)	1.701(.90)	1.648(.90)	L1.716(.90)
64	.7371L	282.85G	2.23	1.686(.90)	L1.662(.90)	L1.703(.90)	L1.648(.90)	1.705(.90)
65	.7289	249.78	2.07	1.561(.80)	1.550(.90)	1.594(.90)	1.554(.90)	1.637(.90)
276	.4879	-17.10	.36+	1.133(.30)	1.114(.30)	1.126(.30)	1.099(.30)	1.145(.30)

CRSP Index Adjusted for Negative Changes in Nominal Industrial Production

n	Hurst	T-Test	D.W.	Vn(0)	Vn(1)	Vn(3)	Vn(6)	Vn(12)
48	.6675	94.31	1.69	L1.627(.90)	1.624(.90)	L1.743(.90)	G1.776(.95)	G1.980(.98)
49	.6709L	100.51L	1.69	1.612(.80)	L1.625(.90)	1.734(.90)	1.756(.95)	1.887(.98)
50	.6598	95.02	1.60	1.523(.80)	1.561(.80)	1.672(.90)	1.701(.90)	1.862(.95)
64	.5668	41.28	.98+	1.386(.70)	1.401(.70)	L1.508(.80)	L1.547(.80)	L1.656(.90)
65	.5676L	42.97L	.99+	L1.415(.70)	L1.412(.70)	1.505(.80)	1.533(.80)	1.589(.80)
66	.5600	37.99	1.01+	1.316(.60)	1.325(.60)	1.403(.70)	1.411(.70)	1.494(.80)
196	.5470	40.79	.26+	1.333(.60)	1.317(.60)	1.387(.70)	1.391(.70)	1.482(.80)

CRSP Index Adjusted for Negative Changes in Real Industrial Production

n	Hurst	T-Test	D.W.	Vn(0)	Vn(1)	Vn(3)	Vn(6)	Vn(12)
47	.5242L	10.08	1.45	L1.558(.80)	L1.592(.80)	L1.632(.70)	L1.675(.90)	1.825(.95)
48	.5239	10.45L	1.45	1.534(.80)	1.560(.80)	1.585(.80)	1.641(.90)	L1.857(.95)
49	.5172	7.83	1.45	1.486(.80)	1.516(.80)	1.559(.80)	1.595(.80)	1.824(.95)
64	.3709L	-56.52	.95+	L1.194(.40)	L1.227(.50)	1.308(.60)	1.339(.60)	1.447(.70)
65	.3633	-60.63	.92+	1.172(.40)	1.205(.40)	1.291(.60)	1.322(.60)	1.432(.70)
66	.3592	-68.16L	.92+	1.190(.40)	1.224(.50)	L1.310(.60)	L1.351(.60)	L1.465(.70)
132	.3168	-139.18	.59+	0.956(.10)	0.978(.10)	1.029(.20)	1.050(.20)	1.107(.30)

CRSP Index Adjusted for Negative Changes in Real M1 Money Supply

n	Hurst	T-Test	D.W.	Vn(0)	Vn(1)	Vn(3)	Vn(6)	Vn(12)
47	.3433	-71.07	1.02+	1.320(.60)	L1.407(.70)	L1.509(.80)	L1.638(.90)	1.900(.98)
48	.3321	-78.43L	1.04+	1.247(.50)	1.323(.60)	1.431(.70)	1.575(.80)	L1.928(.98)
49	.3509L	-68.92	1.20+	L1.354(.60)	1.403(.70)	1.500(.70)	1.618(.80)	1.873(.98)
64	.4131	-54.43L	.94+	L1.247(.50)	L1.293(.50)	L1.356(.60)	L1.413(.70)	L1.535(.80)
65	.4227	-47.55	.88+	1.235(.50)	1.271(.50)	1.336(.70)	1.383(.70)	1.503(.80)
66	.4307L	-42.48	.88+	1.224(.50)	1.259(.50)	1.321(.60)	1.365(.60)	1.458(.70)
154	.6132	87.86	.29+	1.201(.40)	1.263(.50)	1.295(.60)	1.303(.60)	1.351(.60)

G - Global Maximum
L - Local Maxima
+ - Significant positive autocorrelation Durbin Watson Test
* - Indeterminate Durbin Watson Test
Probability in parenthesis is Lo test from Table 1