Long Memory and Indian Stock Market – An Empirical Evidence

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Abstract

This paper makes a serious attempt to explore whether there exists a need to study the use of non-linear models to test the existence of long memory in an emerging market like India. It starts by discussing how various authors are challenging the efficient market hypothesis. This has led to the use of non-linear dynamic systems for modeling movement in stock prices. In order to confirm whether the efficient market hypothesis is applicable to the Indian Stock Market, the study has used the NSE NIFTY returns for the last decade and tested them for normality. Finally, two important tests have been performed using these data: the Variance ratio test and the Rescaled Range (R/S) Analysis to test for persistence in the NIFTY daily returns.

Introduction

According to the Efficient Markets Hypothesis (EMH) an efficient capital market is one in which security prices adjust rapidly to the arrival of new information, and therefore, the current prices of securities reflect all information about the security. Three sets of assumptions imply an efficient capital market: (a) an efficient market requires that a large number of competing profit-maximizing participants analyze and value securities, each independently of the others, (b) new information regarding securities come to the market in a random fashion, and the timing of one announcement is generally independent of others, and (c) the competing investors attempt to adjust security prices rapidly to reflect the effect of new information. Although the price adjustment may be imperfect, it is unbiased. This means that sometimes the market will over-adjust or under-adjust, but an investor cannot predict which will occur at any given time. If we believe that efficient market hypothesis is a valid proposition, then the current asset prices should reflect all generally available information. The efficient market hypothesis implies that since market prices reflect all available information, including the information about the future, the only difference between the prices at P_t and P_{t+1} are events that we cannot possibly predict, i.e. a random event. Hence, in an efficient market, stock prices can be statistically tested for random walk hypothesis.

Recently the efficient markets hypothesis and the notions connected with it have provided the basis for a great deal of research in financial economics. A voluminous literature has developed supporting this hypothesis. Briefly stated, the hypothesis claims that asset prices are rationally related to economic realities and always incorporate all the information available to the market. This implies the absence of exploitable excess profit opportunities. The early survey of Fama (1965) concluded that the stock market was efficient. Fama (1965) analyzed the distribution of a large data set and showed that empirical evidence seems to confirm the random walk hypothesis: a series of prices changes have no memory. The main theoretical explanation that lies behind this observation is the efficient market hypothesis. The EMH has received a lot of empirical support in the academic literature during seventies and eighties. This line of thought has always been received with a lot of skepticism in the professional community, which led to the use of charts and technical analysis rules for trading strategies in markets. Professionals have always claimed that classical statistical tests are mainly linear and therefore, unable to capture the complex pattern of price changes exhibit. However despite the widespread allegiance to the notion of market efficiency, a number of studies have suggested that certain asset prices are not rationally related to economic realities. For example, Summers (1986) argues that market valuations differ substantially and persistently from rational valuations and that existing evidence based on common techniques does not establish that financial markets are efficient.

Time series forecasting is an important research area in several domains. Traditionally, forecasting research and practice has been dominated by statistical methods. As we get to know more about the dynamic nature of the financial markets, the weakness of traditional methods become apparent. In the last few years, research has focused on understanding the nature of financial markets before applying methods of forecasting in domains including stock markets, financial indices, bonds, currencies and varying types of investments.

Although most of the empirical tests of the efficient markets hypothesis are based on linear models, interest in nonlinear chaotic processes has in the recent past experienced a tremendous rate of development. There are reasons for this interest, one of which being the ability of such processes to generate output that mimics the output of stochastic systems thereby offering an alternative explanation for the behavior of asset prices.

As Campbell, Lo and MacKinlay argue (1997): “many aspects of economic behavior may not be linear. Experimental evidence and casual introspection suggest that investors’ attitudes towards risk and expected return are nonlinear. The terms of many financial contracts such as options and other derivative securities are nonlinear. And the strategic interactions among market participants, the process by which information is incorporated into security prices and the dynamics of economy wide fluctuations are all inherently nonlinear. Therefore, a natural frontier for financial econometrics is the modeling of non-linear phenomena”.

Besides its heterokedasticity long-range dependence, long memory process has other certain unique properties. Mandelbrot and Wallis (1969) and Mandelbrot (1972) showed a long-range dependence process could demonstrate itself as a highly non-Gaussian time series with large skewness and kurtosis, and carries non-periodic cycles. A long memory process could allow conditional heteroskedicity (Fung et al 1994), which could be the explanation of nonperiodic cycles. It seems a long memory model is more flexible than an ARCH model in terms of capturing irregular behaviour.

In this paper, I have tried to test the long memory of the Indian stock market using CNX NIFTY as the market proxy. For doing the same I have chosen two tests which are considered robust for testing the long memory component of markets. These two models could complement each other and allow a comparison of the robustness of the results. The paper is organized into the following sections: Section I is dedicated to motivation and objectives of the study, Section II discusses the existing literature on the subject, Section III describes data and the data characteristics, Section IV provides the theoretical background of the models and the justification of their selection, and gives the results of the study and the Final Section gives the conclusions of the study.

Motivation and Objectives: experts on financial markets and economists on both Random Walk and Efficient Market Hypothesis have undertaken good amount of research work. However, long memory models are relatively new to applied economics. Though its origin dates back to Mandelbrot's (1969) work, it was not until 1980's that researchers began to apply the rescaled range analysis, one of the tools in the long memory theory, to financial markets and macroeconomic prices. In 1991 Lo modified the classical R/S method. Based on Beran (1994, pp-41-66), a stationary process with long memory has the following qualitative features:

- Certain persistence exists. In some periods the observations tend to stay at high levels, in some other periods, the observations tend to stay at low levels.
- During short time periods, there seem to be periodic cycles, However looking through the whole process, no apparent periodic cycles could be identified.
- Overall, the process looks stationary.

The primary objective of this study is to investigate if the price behaviour in Indian stock market can be characterized by long memory models. It is not hard to find evidence to argue that price series with random appearance might be non-linear dynamic. But it is difficult to say what kind of non-linear dynamics. Another commonly used stochastic model ARCH and its variants share similar symptoms with long memory models, such as non-normality and heteroscedasticity, but they have totally different generating mechanisms and implications. A time series with ARCH property typically has two components, a conditional mean and a conditional variance function. The non-linearity of the series comes from the non-linearity of the conditional variance. An ARCH model that fits the data well could improve the prediction of the variances of prices but not the price itself (Bera and Higgins 1995). A long memory model is a single mean equation (system) and has a flexible structure. It represents short as well as long memory simultaneously.

Literature available depicts studies concerning developed and emerging markets, though major work has been undertaken for developed market like US. The unavailability of reliable data may one of the important reasons why very few studies have been undertaken in emerging markets. For Indian financial market very few studies have tested long memory component. Some important work has been done by Barman and Madhusoodan (1993), Thomas (1995) and Basu & Morey (1998) covering Indian market. Basu & Morey used Variance Ratio Test methodology devised by Lo and MacKinlay to test the long memory of the Indian stock market. The current proposal is to study Indian stock market with respect to its long-memory using stock market returns during last one decade during which the stock market has gone through the liberalization process and also on few occasions it has been subject to few extreme volatile situations for many reasons. The study attempts to examine the efficient market hypothesis on Indian conditions by implementing two important techniques that are robust to time varying volatility. The study has been based on the idea that variance keeps changing over time and hence a test like Variance Ratio test would not only help to test the random walk theory in stock prices but also being robust to time-varying volatility.

The potential presence of stochastic long memory in financial asset returns has been an important subject of both theoretical and empirical research. If assets display long memory, or long-term dependence, they exhibit significant autocorrelation between observations widely separated in time. Since the series realizations are not independent over time, realizations from the remote past can help predict future returns. Persistence in share returns has a special claim on the attention of investors because any predictable trend in returns should be readily exploitable by an appropriate strategy by the market participants.

During last one decade, the Indian financial system has been subjected to substantial reforms with far reaching consequences. These reforms process has helped in dramatic improvement in transparency level in financial markets including stock market. The regulatory changes that have taken place during last one decade of financial sector reforms has led us to believe that financial market have become more efficient with respect to the price discovery mechanism and helped the market to grow exponentially. The country has also experienced the mild contagion effect of financial crisis in International markets and successfully sailed through the period of Asian crisis not significantly jeopardizing the interest of the domestic economy. There have been significant changes in the regulations for smooth and efficient functioning of capital market in the country. During the last decade we have seen cleansing of the stock market system by market regulators and emergence of National Stock Exchange of India has greatly helped the system to achieve the present level of transparency and efficiency. The market has undergone substantial change due to introduction of hedging products like futures and options. Risk management system has been changing in keeping pace with change in scenario. Other reforms in the form of deregulation of interest rate, tax reforms, banking sector reforms, reforms in the external sector, etc. has also helped market participants to value assets according to their intrinsic values. Liquidity has greatly increased as the market spread has reached the far away villages bringing investors together. The concept of developing a large order book in the stock market made the pricing of stocks more accurate and efficient and also resulted in bringing down the bid/ask spread benefiting the investors community as a whole. International investors' access to the domestic market has also helped in increasing liquidity. All these helped in better dissemination of information and hence possibly increased the level of efficiency in asset prices. The level of such efficiency in prices need to be tested with various models that exist in the literature. The earlier work done on Indian market (Basu and Morey (1998) and Barman and Madhusoodan (1993), Thomas (1995) that has a major component from the pre-reforms period. Reforms process has led to a regime shift and hence it is necessary to test the market with the market data after the introduction of financial sector reforms.

Literature Review:

Helms (et al 1984) applied rescaled range analysis to detect the existence of long memory in the futures prices of the soyabean complex. With the Hurst exponent in the range of 0.5 to 1 indicating long memory, the authors found that the Hurst exponents ranges from 0.558 to 0.71 for daily prices of two futures contracts in 1976 and from 0.581 to 0.67 for intra-day prices of 5 contracts in 1977-78. Fung and Lo's (1993) long memory study analyzed the prices of two interest rate futures markets, Eurodollars and Treasury Bills. The results from the classical R/S analysis and Lo's (1991) modified R/S analysis provide no evidence of the existence of the long memory and support for the weak form EMH. Peters (1994) notes that most financial markets are not Gaussian in nature and tend to have sharper peaks and fat tails, a phenomenon well known in practice. One of the key observations made by Peters (1994) is the fact that most financial markets have a long memory, what happens today affects the future forever. One strand of my motivation comes from Peters, (1994) Fractal Market Hypothesis. Long memory analysis have been conducted for stock prices (Greene and Fielitz (1997), Aydogan and Booth (1988), Lo (1991), Cheung, Lai and Lai (1993), Cheung and Lai (1995), Barkoulas and Baum (1997)), spot and futures currency rates (Booth, Kaen and Koveos (1982a) Cheung (1993a), Cheung and Lai (1993), Bhar (1994), Fang, Lai and Lai (1994), Barkoulas, Labys and Onochie (1997a)), gold prices (Booth, Kaen and Kovoes (1982b)), Cheung and Lai (1993)), international spot commodity prices (Barkoulas, Labys, and Onochie (1976b)), and commodity and stock index futures (Helms, Kaen and Koveos (1984), Barkoulas, Labys, and Onochie (1997a)), inflation rate (Scacciavillani (1994), Hassler and Wolters (1995), spot and forward metal prices (Fraser and MacDonald (1992)). Fung et al (1994) considered intraday stock index futures and tested for long memory by using variance ratio, R/S and AFIMA models. All these types of analyses concluded that no long memory exists in the data.

The results are mixed, but all the authors agreed that identification of long memory is very significant in at least two senses: (a) the time span and strength of long memory will be an important input for investment decisions regarding investment horizons and composition of portfolios; and (b) prediction of price movements will be improved. On the background of this, it has become important to test the long memory existence in Indian market taking the stock market data for last one decade during which substantial regulatory changes have taken place and the market practices have changed dramatically and various hedge products have been introduced to improve the risk management.

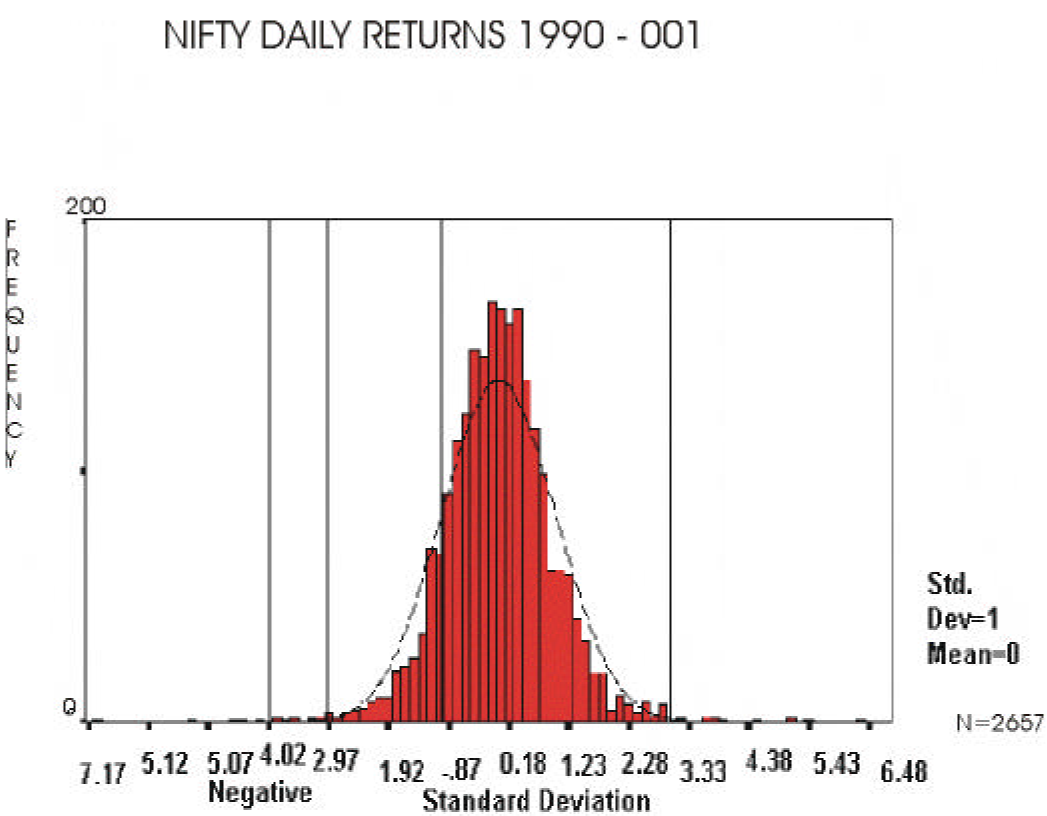
Data and Data Characteristics:

The procedures for collecting and transforming data affect any serious statistical modeling. The daily closing values of the index NIFTY for the period from July 1990 to November 2001 is considered for the study. From July 1990 to October 1995, the NIFTY values used here is the simulated values maintained by IISL (a subsidiary of NSEIL which looks into the Index products of NSEIL). From November 1995 to November 2001, the actual close values of NIFTY has been taken for the purpose of the study. The index consists of underlying stocks whose closing prices determine the closing values of NIFTY. But using daily prices often encounters one problem; the limits for daily price changes, based on the closing price of the previous day. Therefore the series are truncated and that might distort non-linear modeling. Due to earlier provision of price bands, many times the stocks hit a circuit breaker and hence the series gets distorted. What we have today is not the price bands on the individual stocks, but there is a band for the index. However, the analysis of daily price data is necessary to understand any findings. Returns have been calculated for various time lags like 1 day, 14 day, 30 day, 90 day, 180 day, 270 day, 360 day, 720 day and 1800 days to understand to what extent, the long memory process exists, if it exists at all.

Tests of Normality of the Data

Sharpe (1970) notes that: "normal distributions assign very little likelihood to the occurrence of really extreme values. But such values occur quite often." Turner and Weigel (1990) (as quoted in Peters, 1996) in an extensive study of volatility, using S&P index returns from 1928 through 1990, found that "daily return distributions for the Dow Jones and S&P are negatively skewed and contain a larger frequency of returns around the mean interspaced with infrequent very large and very small returns as compared to a normal distribution."

While analyzing the CNX NIFTY returns data for the period from July1990 to November 2001 it was found that the tails were a bit fatter, and more significantly the peak around the mean was higher than predicted by the normal distribution. This can be seen from the graph below. The daily returns were normalized so that they have a mean of zero and standard deviation of one. The most common explanation of the fat tails is that information shows up in infrequent clumps, rather than in a smooth and continuous fashion.



What we can also see from the above, at both the tails, we have incidence of three and four sigma event. The daily returns are negatively skewed for NIFTY and contain a large frequency of returns around the mean.

The following table summarises the findings for NIFTY:

	Average	**Std Dev**	**Skewness**	**Kurtosis**
1990-2001	0.0507	1.8985	***-0.0068***	***4.5839***
1990-95	0.097	1.961	***0.0444***	***5.7522***
1996-01	0.010	1.769	***-0.0970***	***2.4051***

Now let us concentrate to see if the trends indicate the persistence of long memory in NIFTY returns using two important methodologies.

Persistence Tests

If asset prices display long memory, or long-term dependence, they exhibit significant autocorrelation between observations widely separated in time. This implies that what has happened not only in recent past but long time back has a bearing on the today's market prices and hence existence of an autocorrelation between these observations. Today's risk containment policies followed in Indian securities market is built on the basis of historical price behaviour. Since the series realizations are not independent over time, realizations from the remote past can help us predict future movements in asset prices. Persistence in share returns has a special claim on the attention of investors because any predictable trend in returns should be readily exploitable by an appropriate strategy.

A number of studies have tested the long memory hypothesis for stock market returns. Peters (1989) used Hurst Rescaled Range (R/S) analysis to measure non-periodic cycles in financial data. He concluded that capital market prices do not reflect information immediately, as the efficient market hypothesis assumes, but rather follow a biased random walk that reflects persistence. Using the rescaled range (R/S) method, Greene and Fielitz (1977) also reported evidence of persistence in daily U.S. stock returns series.

Barkoulas and Baum used Spectral Regression Test to test the long memory of US stocks and found only few stock do have long memory.

Related research into stock market overreaction has also uncovered evidence that a measure of predictability can be identified ex post in stock returns. Specifically, shares of companies, which have performed well in the past subsequently, perform less well in the future, while shares, which have performed badly in the past usually improve their performance (MacDonald and Power, 1992).

However, according to some authors, the classical R/S test is biased toward finding long-term memory too frequently. Stock market returns may follow biased time paths that standard statistical tests cannot distinguish from random behavior. Rescaled range analysis can be used to detect long-term, non-periodic cycles in stock market returns. If this technique is not applied correctly, however, then it can be influenced by short-term biases, leading to the erroneous conclusion that the stock market has long-term memory.

Lo (1991) developed a modified R/S method, which addresses some of the drawbacks of the classical R/S method. Using the variant of R/S analysis, Lo finds no evidence to support the presence of long memory in U.S. stock returns. Applying Lo test, which does not rely on standard regression techniques and is robust to short-term dependence, provides statistical support for the hypothesis that stock market returns follow a random walk (Ambrose, 1993). Using both the modified R/S method and the spectral regression method, Cheung and Lai (1995) find no evidence of persistence in several international stock returns series. Crato (1994) reports similar evidence for the stock returns series of the G-7 countries using exact maximum likelihood estimation. Wright (1999) used AFRIMA model to test long memory in emerging market including India and came up with the conclusion that emerging markets appear to have considerable serial correlation which stands contrast to the results for the developed markets like US, where there is little evidence for any serial correlation in stock returns.

The primary focus of these studies has been the stochastic long memory behaviour of stock returns in major capital markets. In contrast, the long memory behaviour in smaller markets has received little attention. Contrary to findings for major capital markets, Barkoulas, Baum, and Travlos in a Working Paper found significant and robust evidence of positive long-term persistence in the Greek stock market.

Today, we see an overwhelming response to emerging markets from investors across the world. These markets have provided diversification opportunity to international investors. It must be noted that such markets are very likely to exhibit characteristics different from those observed in developed capital markets as the market micro-structure is different in emerging markets vis-à-vis developed ones. Biases due to market thinness and non-synchronous trading should be expected to be more severe in the case of the emerging markets. In case of Indian stock market, we have seen on many occasions irregularities in price behaviour not because of the true market conditions but due to some sort of manipulations carried out by a group of greedy market participants to throw the market out of gear. Another important factor that need to be considered is that these emerging markets have been going through many regulatory changes to improve the efficiency level and can not be fully compared with the developed and established markets.

In this study, I look for evidence of long memory in Indian capital market. I have used data about returns from the National Stock Exchange of India Ltd. to check for persistence of long memory in daily returns data. The Nifty data is actual closing values from November 1995 to November 2001 while the data pertaining to the period before November 1995 has been simulated NIFTY data using the closing prices of the related stocks that might have been obtained from other stock exchanges before NSEIL started operation in November 1994. These dataset has been constructed by NSEIL while building NIFTY and the same is available with IISL Two statistical methods have been used for the test: (a) the Variance Ratio Test and (b) the Hurst Exponent (R/S Analysis) to test the data. These two tests have been selected to find out if both give the same result or the results differ even if we use the same dataset.

Variance Ratio Test

The variance ratio test has been used to test permanent as well as the temporary memory components. Barman & Madhusoodan (1993) carried out the test for Indian market to find out the long memory.

Variance Ratio Test popularized by Cochrane (1988), and used by MacDonald and Power (1992) etc. can be explained as below:

$$Z(k) = (1/k) * \{ Var(X_{t-k}) / Var(X_t) \} \qquad \ldots\ldots..(1)$$

Where X_t denotes a one period return, obtained from the first difference of the natural logarithmic of the share price, and X_{t-k} denotes the k-period return calculated using the kth difference of the share price. The possibilities are as follows:

1. If the price series follows a random walk, this ratio should equal unity.
2. If the series is stationary, the ratio will tend to zero.
3. If share price exhibit mean reversion, Z (k) should lie between zero and one.
4. Values of Z (k) above one indicate that a current increase in the value of the share price will be reinforced in the future by further positive increases.

Performing the above analysis on the data of more than 10 years of NIFTY closing values, following results were obtained:

Lag	**1 Day**	**14 days**	**30 days**	**90 days**	**180 days**	**270 days**	**360 days**	**720 days**	**1800 days**
Variance	3.60	39.30	81.26	237.80	379.38	613.74	1200.66	1073.12	883.29
Variance Ratio		***0.7789***	***0.7516***	***0.7331***	***0.5848***	***0.6307***	***0.9254***	***0.4135***	***0.1362***

The above tests show some interesting results. The above results show that in the short as well as long term the variance ratio has been less than 1 that indicates there is a definite mean reversion tendency for the Indian stock market if we consider NIFTY as the market proxy. But for the time lag of 5 years, it is moving towards 0. The interesting point to note here is that for the lag of 360 days, we have the Variance Ratio that is close 1.

R/S Analysis

H, a Hurst exponent, is produced by the rescaled range analysis, or R/S analysis which was established by hydrologist H E Hurst in 1951, further developed by B Mandelbrot in the 1960s and 1970s, and applied to economic price analysis by Booth et al (1982), Helms et al (1982) Peters (1989) and others in 1980s. For a given time series, the Hurst exponent measures the long-term nonperiodic dependence and indicates the average duration the dependence may last. Standard autocorrelation tests detect long-term dependency in stock market prices if dependent behaviour is periodic and if the periodicity is consistent over time. Fundamental historical changes however alter the period of cycles. Mandelbrot (1972) proposes a statistic to measure the degree of long-term dependency, in particular, “non periodic cycles”. The rescaled range, or R/S statistic, is formed by measuring the range between the maximum and minimum distances that the cumulative sum of a stochastic random variable has strayed from its mean and then dividing this by its standard deviation. An unusually small R/S measure would be consistent with mean reversion, for instance, while an unusually large one would be consistent with return persistence. To construct this statistic, consider a sample of returns $X_1, X_2, \ldots, X_n$ and let X denote the sample mean.

$$Q_n = 1/(\sigma_n . \sqrt{n}) \left[\max \Sigma (X_j - X_n) - \min \Sigma (X_j - X_n) \right] \qquad \ldots\ldots.(2)$$

In his original work, Mandelbrot suggested using the sample standard deviation estimator for the scaling factor, σ_n

We have used this as my second technique to judge persistence in stock market returns. Peters (1994) has discussed this method in a simpler and neater way. Let us take a series of data $X_1, X_2, \ldots, X_n$ and let X denote the sample mean. Let σ_n again be the standard deviation. The rescaled range was calculated by first rescaling or “normalizing” the data by subtracting the sample mean:

$$Z_r = X_r - X \qquad r = 1,2\ldots\ldots n \qquad \ldots\ldots\ldots (3)$$

The resulting series, Z, now has a mean of zero. The next step creates a cumulative time series Y:

$$Y_1 = Z_1 + Z_r \qquad r = 2,3\ldots\ldots n \qquad \ldots\ldots\ldots(4)$$

Note that by definition the last value of Y (Y_n) will always be zero because Z has a mean of zero.

The adjusted range, R_n is the maximum minus the minimum value of the Y_r:

$$R_n = \max(Y_1, \ldots\ldots\ldots, Y_n) - \min(Y_1, \ldots\ldots\ldots, Y_n) \qquad \ldots\ldots..(5)$$

The subscript n for R_n now signifies that this is the adjusted range for X_1, X_2, ….., X_n. Because Y has been adjusted to a mean of zero, the maximum value of Y will always be greater than or equal to zero, and the minimum will always be less than or equal to zero. Hence, the adjusted range R_n will always be non-negative.

However, this equation applies only to time series in Brownian motion: that have mean zero and variance equal to one. To apply to any time series (like stock returns), we need to generalize the equation. Hurst found that the following was a more general form of equation:

$$R/\sigma = c * n^H \qquad \ldots\ldots\ldots(6)$$

The R/S (or R/σ) value is referred to as the rescaled range analysis because it has mean zero and is expressed in terms of local standard deviation. In general, the R/S value scales as we increase the time increment, n, by a "power–law "value equal to H, generally called the Hurst exponent.

The procedure used for calculations is listed below (Peters, 1994; pp. 62-63).

1. Begin with a time series of length M. Convert this into a time series of length N = M – 1 of logarithmic ratios: $N_i = \log(M_{i+1} / M_i)$, I = 1,2,3, …….., (M-1)

2. Divide this time period into A contiguous sub periods of length n, such that A*n = N. Label each sub period I_a with a = 1,2,3,….A. Each element in I_a is labeled $N_{k,a}$ such that k = 1,2,3,…n. For each I_a of length n, the average value is defined as: $e_n = (1/n) * \Sigma N_{k,a}$ where e_n = average value of the Ni contained in sub period I_a of length n.

3. The time series of accumulated departures ($X_{k,a}$) from the mean value for each sub period I_a is defined as $X_{k,a} = (N_{i,a} – e_a)$ where k = 1,2,3, ….n

4. The range is defined as the maximum minus the minimum value of $X_{k,a}$ within each sub period I_a: $R_{Ia} = max(X_{k,a}) – Min\ (X_{k.a})$ where 1<= k<= n

5. The sample standard deviation calculated for each sub period I_a: $S_{Ia} = (1/n) * \Sigma(N_{i,a} – e_a^{\,2})^{0.50}$

6. Each range R_{Ia} is now normalized by dividing the S_{Ia} corresponding to it. Therefore, the rescaled range for each I_a sub period is equal to R_{Ia} / S_{Ia}. From step 2, we had A contiguous sub periods of length n. Therefore, the average R/S value for length n is defined as $(R/S)n = (1/A) * \Sigma\ (R_{Ia} / S_{Ia})$

7. The length n is increased to the next higher value, and (M-1) /n is an integer value. We use values of n that include the beginning and ending points of the time series, and steps 1 through 6 are repeated until n = (M-1)/2.

Hurst's Empirical Law

Hurst (1951) also gave a formula for estimating the value of H from a single R/S value (as quoted in Peters, 1996):

$$H = \log (R/S) / \log (n/2) \qquad …….(7)$$

where n = number of observations

This equation assumes that the constant c of the above equation is equal to 0.5. Feder (1988) shows that the empirical law tends to overstate H when it is greater than 0.70 and understate it when it is less than or equal to 0.40. However for short data sets, where regression is not possible, the empirical law can be used as a reasonable estimate. It is clear that H is a parameter that relates mean R/S values for subsamples of equal length of the series to the number of observations within each equal length subsample. H is always greater than 0.

The method discussed above would become clearer by looking at the calculations done for NIFTY data. We can use daily NIFTY closing data for the last decade (i.e. 1990 to 2001) and calculate daily logarithmic returns. The same data has been used to estimate H for 8 periods of time (with N=2, 30, 90, 180, 270. 360, 720, & 1800) since plotting a line between log (R/S) vs log (N) is very difficult.

There are 3 distinct classifications for the Hurst exponent (H):

1. H = 0.5

H equal to 0.5 denotes a random series, the process is white noise.

2. 0<=H < 0.5

This type of system is anti persistent or mean reverting. That means if the system has been up in the previous period, it is likely to be down in the next period. The strength of anti-persistent behaviour will depend on how close H is to 0.

3. 0.5 < H <1.0

Here we have a persistent or trend reinforcing series, long memory structure exists. That means, if the series has been up (down) in the last period, hence the chances are that it will continue to be positive (negative) in the next period. Trends are apparent. The strength of the trend-reinforcing behaviour, or persistence, increases as H approaches 1. The closer H is to 0.5, the noisier it will be and the trend would be less defined. Persistent series are fractional Brownian motion, or biased random walks. The strength of the bias depends on how far H is above 0.50. The greatest advantage of R/S analysis is that the measure is independent of the distribution assumption for a given series. The robustness of results remains unaffected regardless whether the distribution is normal or non-normal. The dependence the Hurst exponent captures is the nonlinear relationship inherent in the structure of the series (Peters (1991)).

The results for the NIFTY results are listed below:

Period, N	***14 days***	***30 days***	***90 days***	***180 days***	***270 days***	***360 days***	***720 days***	***1800 days***
Hurst Exponent	***0.6130***	***0.6223***	***0.6261***	***0.6211***	***0.6084***	***0.5825***	***0.5941***	***0.5341***

None of the values for the time lags is equal to 0.50 indicating that Indian stock market can not be said to follow random walk in so far as the daily returns are concerned when we use NIFTY as the market proxy. This shows that there is a definite possibility for persistence in the NIFTY returns but the values for time lags above 360 days are very close to 0.50 leading us to believe that there is enough noise in the series and the trend is not perfectly established. However, for the shorter period (up to 9 months time lag), the values are reasonably higher than 0.5 indicating a definite possibility for persistence.

Conclusion

The normality tests on the daily NIFTY returns for the last one decade indicates the need to explore the application of non-linear modeling techniques in capital market. But we come to see that the results from the persistence tests are split. The variance test clearly implies that there does not exist any short-term or long memory in the market returns as given by NIFTY returns data for more than 10 years and it shows a clear pattern of mean-reversion. However, the R/S analysis does give indications of long-term memory for all time lags but with higher time lags of more than two years there exists long memory but with noise as the values are close to 0.5. In either case, analysis shows that the movement of stock prices does not follow a random movement. However, a more rigid analysis needs to be performed, maybe by using Lo's modified R/S Analysis. Also, for a foolproof analysis, the data used should be for a period longer than just one decade. Studies need to be undertaken on individual stocks to understand if the component stocks of the proxy also follow the same pattern and in case they do not follow the trend as given by either Variance ratio test or R/S analysis for NIFTY, what could be the possible explanation. These studies need to be conducted to understand if the present risk containment system applicable in capital market margining is in line with the long memory of the capital market. If there is not enough proof to support long memory, then we can discard the path of historical volatility to use as margining principles and go for something else.

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