H30

Global (long-term) dependence in economics and finance (long ⋆foreword and excerpts from M 1969e, M 1971n, and M 1972c)

•⋆***Chapter foreword.*** Links between this book and two earlier volumes of these *Selecta* are described in Chapter ⋆H1 by a "phase diagram" that acts as a map. Another more historical and more chatty link is provided by this chapter, which is written to be more or less self-sufficient.

For better or worse, each *Selecta* book has a complicated structure, none more than M 1997E. In addition, that book did not signal the end of my involvement with economics and finance, but a strong acceleration already marked by M 2001a, M2001b, M2001c, M2001d.

A popular presentation of what is newest in M 1997E is M 1999s. Its technical substance is elaborated upon in Chapter ⋆H1.

The odd structure of M 1997E resulted in part from a sudden turn of events. For decades, the M 1963 model of price variation was criticized for all the wrong reasons, beginning (in Chapter E17) with the editor's comments in Cootner 1964. Suddenly, my findings became widely accepted and started attracting wide attention; this is why M 1997E was rushed out.

Criticism was also addressed to the M 1965 model's assertion that economics and finance are filled with examples of global dependence. To take one influential example, Lo 1991 criticized the evidence of *R/S* analysis as being overly dependent on graphics – while objective statistical tests find global dependence to be present in some cases but absent in many others, with no rhyme or reason. Lo left the reader with the familiar impression that formalism can always be trusted, while graphics is misleading. But, once again, this impression was ill-founded and the correct conclusion is the precise opposite. Lo's tests may be appropriate for Gaussian processes, but the only justification for their use in finance is the lack of anything better.

The fact that they yielded meaningless results illustrates again two themes that this book keeps hammering. First, one *must not* study wildly variable phenomena by using tools addressed to mild variability. Second, every use of an old tool in a new context tests the structure of the context but also tests the tool.

Be that as it may, power-law relations and scaling kept being bashed (see Chapter H8) until, once again, my findings suddenly became widely accepted. Today, papers on fractional Brownian motion (FBM) appear all over.

As a result, the number of pages this book devotes to economics and finance cries out to be increased – but must be limited to this brief chapter.

Background for the reader unfamiliar with the problem of modeling the variation of financial prices; discrepancies between Wiener Brownian motion (WBM) and every aspect of the evidence. The standard model of financial price variation is WBM. Once again, it is a random walk in which each step (a) is statistically independent of all past steps and (b) follows the Gaussian distribution. Both assumptions are ancient (1900) and convenient, but neither agrees with the evidence.

For example, M 1963b{E14} is best known for focusing on non-Gaussianity and long tailedness, and advancing the "M 1963 model," technically called "mesofractal." That model postulates that price increments are independent but follow a Lévy stable distribution. However, even that paper began and ended with comments on serial dependence.

The back half of M 1997E reproduces old papers that center around the mesofractal M 1963 model. The front half, to the contrary, reports on totally new work centering around my latest "M 1972/97" model based on multifractals: in this model, price is an FBM in multifractal trading time.

Only very sketchily does M 1997E mention an intermediate model concerned with global dependence. I call it "unifractal" or "M 1965," because the first reference on its account was M 1965h{H9}. It faced dependence and pioneered the use of *R/S* analysis in all fields. Its relevance to economics and finance is discussed in Sections 1 to 3 of this chapter, using excerpts from M 1969c, M 1971n, and M 1972c. Another relevant text, in French, is M 1973j{FE}.

A careful reading of those excerpts and the fact that the M 1965 model was underplayed in M 1997E both reveal a high level of discomfort. FBM had proved useful in hydrology but it was, and remains, an overly special Gaussian example of global (long-term) dependence. Finance demanded a specific alternative to FBM endowed with long tails. It was assumed to exist but in the 1960s could not be identified.

For the model-maker, this created a serious quandary. In many securities I studied, the global dependence indicated by *R/S* was extremely strong. However, an FBM with high H would be very highly persistent, hence allowing a high degree of prediction. It would invite arbitrating and, ultimately, the replacement of FBM by a different process.

Unfortunately, as I frequently complained in the following excerpts, no *natural* example known to me before 1972 was at the same time globally dependent and long-tailed. As a result, against my own consistent preaching and practice, I found no way to proceed from the *R/S analysis* of prices to the *synthesis* that had been so useful in hydrology.

This quandary was strongest before I conceived multifractals, and the last short paragraph of M 1972j{N14} gave the first hint of the specific multifractal M 1972/1997 model (see M 1997E, page 42). From the availability of the multifractal alternative, it follows that, today, economics and finance must be sharply distinguished; FBM may be arguably applicable to the former but not to the latter.

The discovery of global dependence and the difference between the forms it takes in economics and finance. What follows is written in terms of the *hierarchy of levels of knowledge* that is described in Chapter H7, Section 0. The reader unfamiliar with that hierarchy may want to read that section first.

Scaling global dependence entered into hydrology through M 1965h{H9} and Adelman 1965 independently reported that the spectral density of economic time series is (overall) monotone decreasing. Though largely qualitative, this important observation instantly made me think of my own work, namely, of global correlation that follows a power law.

The next step was to extend from hydrology to economics a phenomenological approach based on self-affinity pioneered in M 1965h{H9}. The first mention of global dependence in economics occurred in two texts sent to the editors in 1966. One, dated March 1, led to M & Van Ness 1968{H11}; the other, dated May 31, led to M 1969e. That second paper is too long and rambling to be reprinted in full anywhere in these *Selecta*. But it is excerpted in Section 1 of this chapter – and also in M 1999N, in an appendix to Chapter N9. My collaborations with John Van Ness and James R. Wallis are documented in Sections 4.2 and 4.3 of Chapter H8.

Use of R/S analysis to verify the existence of global dependence in financial prices. While FBM itself was misleading, as already mentioned, the tests that showed global dependence in prices became quite influential and even useful. That study began when a Columbia University student in statistics, Murad S. Taqqu, became my programming assistant. I assigned him to write a program to evaluate *R/S* (an easy task today, but a daunting one then). Over a memorable long Christmas weekend when the largest mainframe at IBM Research was all ours, I asked Taqqu to run that program on a large number of price records of securities traded in the New York Stock Exchange. A boxful of nearly faded outputs from a Calcomp pen tracer was (last time I looked) stored in my office. (Taqqu then wrote under my supervison a mathematical statistics PhD thesis on global dependence; this work, now classic, is referenced in Chapter N9 of M 1999N.)

The main conclusions drawn from those tests were reported in several near-identical documents; excerpts are found in Section 2 of this paper. The announced full paper on the results of *R/S* analysis of financial prices never materialized.

Altogether, in terms of work and publications, the exploration of the uses of *R/S* in finance/economics was far briefer than either planned or desirable. While difficulties were arising in interpreting *R/S*, opportunities were beckoning in the physical sciences; they soon led to multifractal measures and then (in a possibly irrational but historically accurate sequence) to fractals other than graphs of random functions. This general situation also explains why the hint at the end of M 1972j{N14} – which was already mentioned, and in due time led to the M 1972/97 model – was not immediately explored.

Effect of the M 1965 model on academic communities in economics and finance: the work of Clive Granger. Very quickly and despite inadequate publication, my views and findings on global dependence in economics and finance reached academia. This happened well before the recent revival marked by M 1997E and M 1997FE. I am far from thoroughly acquainted with this literature but a few comments are in order.

Many economists (including some who mistakenly criticized my work on the basis of inapplicable statistical tools) learned of FBM and global dependence from M 1972c.

Others found out indirectly. The title of Granger 1966, "The typical spectral shape of an economic variable," was rightly honored by becoming a cliché. But its text basically confirmed an already mentioned observation in Adelman 1965.

Later, however, Granger 1978 ended with an after-thought referring to my work, and Granger & Joyeux 1980 observed that "the fundamental reasoning underlying the long-memory models is quite different in [my] papers than [in theirs]. The models that arise are not identical in *details* ... However, it should be emphasized that many of the results to be reported have *close parallels* in this previous literature." (The italics are mine.) In Granger's later publications, the parallelism became complete, though he prefers a discrete-time version of FBM that differs a bit from the Type I and Type II algorithm in M & Wallis 1969a{H13}.

Discretization is usually motivated by unquestionable convenience, but I view it as more than a detail. I favor very heavily the models that possess properties of time-invariance or scaling. In these models, no time interval is privileged by being intrinsic. In discrete-time models, to the contrary, a privileged time interval is imposed nonintrinsically.

Comment on FIGARCH. Space is lacking for references and specialized terms must remain unexplained. Among many other weaknesses, a popular price variation model called GARCH assumes a purely local form of dependence. An effort to be more realistic grafted on GARCH a global (long dependent) correlation patterned after FBM. This graft's lack of elegance would not have mattered if it had been effective. But standard "objective" tests were unconvincing. More importantly, Fisher, Calvet, & M 1997 produced synthetic records of FIGARCH: in no way do they look like the actual data. •

1. EXCERPTS FROM M 1969E: "LONG-RUN LINEARITY, LOCALLY GAUSSIAN PROCESSES, H-SPECTRA, AND INFINITE VARIANCE"

This paper [received by the editors on May 31, 1966)] describes two divergence hypotheses [the author] introduced to account for the main erratic aspects in the behavior of economic time series.

MOST ECONOMIC TIME SERIES do not fulfill the independent Gaussian ideal of simplicity. Discrepancies can be divided into two classes. The first includes local or "high frequency effects," to which the bulk of econometrics has so far been devoted. An example ... is the existence of nonvanishing correlations between successive or nearly successive values of time series. The second includes global or "low frequency effects," with which my own past and present work is concerned...

A common symptom in economics is the presence of "persistence." This is defined as a "tendency" for large (positive or negative) values to be followed by large values of the same sign, in such a way that economic time series seem to go through a succession of "cycles." "Long cycles" are those of wavelength of the order of magnitude of the total sample size T (say, $T/5$ to $T/3$). This picture of cyclic behavior is complicated by the fact that the long cycles seen in different samples of the same time series have different wavelengths, so that different samples *do not* look alike even from the viewpoint of the dominating cyclic behavior.

Adelman 1965 investigated long cycles by spectral analysis... Experience suggests that... the most clearly marked spectral maximum is typically located very near the smallest observable frequency, namely, $1/T$. .... Granger 1966 confirms Adelman's observation. Frequently, the empirical spectral density decreases steadily beyond this maximum. ...

The absence of "factual" long cycles in economic time series leaves no explanation for the fact that for every economic time series, irrespective of its length T, the empirical spectrum has a maximum near $f=1/T$. For reasons I discuss below, I think it best to explain the universality of this shape by *assuming that the spectral density $S'(f)$ of the generating process tends rapidly towards infinity as $f \to 0$*. This assumption is a way of expressing that the span of interdependence of the generating process is infinite.

This requirement ... includes the functions $S'(f)$ that satisfy, for small f, the relation $S'(f) \sim f^{\alpha-2}$, with $1 < \alpha < 2$ . ...

Processes in which $S'(f) = f^{\alpha-2}$, with $1 < \alpha < 2$, have been called fractional noises in M & Van Ness 1968{H11}.... The present paper will be concerned exclusively with processes that are, in a sense to be made precise, "nearly Gaussian." ...

Discussion of the use of "infinity" in the statement that $S'(f) \to \infty$ as $f \to 0$. A set of T readings... can always be considered as generated by a Markov process with a memory that is finite and at most equal to T .... As T increases, however, the estimated value $S'(0)$ will also have to increase. Thus, the estimated generating process depends upon the duration of the available sample. Such estimates... teach us... that interdependence in economic time series extends up to some unknown time span longer than any currently available sample size. It is best, I think, to regard such spans as "infinite for all practical purposes."...

A familiar example of the... concept of "physical infinity." ... Consider the sum S_{100} of 100 binomial random variables, each equal to –1 or 1 with equal probabilities. This sum is surely bounded by –100 and 100, but it is all but universally approximated by an *infinite span* random variable, namely the Gaussian. ...

The "infinite variance syndrome". A second common symptom in... the time series of price changes is the extreme size of the few largest observations in a sample, and the great variability of these sizes between samples. [The] "outliers" make different samples of the same series appear different from each other. Moreover, the sample variances are enormously influenced by the precise values of the outliers; attempts to estimate "the" population variance end in grief.

To account for the erratic variability of sample variances without giving up stationarity,... claim that the population variance of the generating process is infinite. ....

Discontinuity. ... Infinite variance and sample function discontinuity are linked logically. They are also linked empirically, since speculative price series are capable of sharp and practically instantaneous variation.

The "Noah" and "Joseph" effects. The $1/f$ spectrum and infinite variance syndromes are widespread in the physical sciences. The comments made on this account in M 1963e{E3} have been confirmed, and have even turned out to be conservative. The Biblical stories of Joseph and of Noah show that persistence and the presence of outliers has long been in the context of water levels. This phenomenon directly affects the production and price of agricultural commodities. Until recently, however, hydrology and economics both concentrated on high frequency problems. Among the exceptions is M 1966b, a study of the relation between weather and price to which the present work is intimately related... Persistence and outliers are also extremely widespread [in] electronics....

The study of the "Joseph Effect" of hydrology is immediately translatable into considerations about the $1/f$ spectrum hypothesis of economics.

A limitation of the linear models of the economy. The hypotheses of $1/f$ spectrum and infinite variance [demand] vectorial random processes in which some coordinate processes have an $1/f$ spectrum and smooth (continuous) sample functions while other coordinate processes have an infinite variance and very jumpy (discontinuous) sample functions.

Such coexistence is inconceivable within the framework of traditional econometric models – linear with Gaussian error terms....

The model I shall describe is not "linear," except "in the long-run."

2. EXCERPTS FROM M 1971n: "STATISTICAL DEPENDENCE IN PRICES AND INTEREST RATES"

As an invited speaker at the Second World Congress of the Econometric Society (Cambridge U.K., 1970), I distributed a handout, M1970e, that was split for publication between two parts in the 1970 and 1971 Annual Reports of the National Bureau of Economic Research, *an influential institution with which I was associated at the time, to mutual benefit. One of those short pieces consisted in dry descriptions of R/S analysis that need not be repeated here.*

VARIOUS FINANCIAL PRICES, NAMELY, interest rate series in Macaulay 1936, prices of commodities from various sources, and series of daily and monthly returns on securities were examined by R/S analysis and found to fall in different categories.

Certain prices, including the interest on call money, exhibit strong persistence say $J=0.7$. This was to be expected, because call money is a tool of arbitrating, so its price cannot itself be arbitrated; its behavior should follow closely that of the various exogenous economic quantities. There is strong evidence that economic time series other than price changes and various physical (e.g., climatic) triggers of the economy are strongly persistent, so the observed behavior of call money rates was as expected.

At the other extreme, certain prices, including those of British Consols, cash, wheat, and some securities, have *R/S* independent increments. The reason for this is unclear. They are possibly dominated by what may be called "market noise." Spot commodity prices are not subject to thorough arbitrating, however, so the absence of persistence in wheat is something of a puzzle. Explanation may be sought in institutional features; the arbitrating present in future price may have an indirect effect on spot prices.

The intermediate cases are those where there is a slight dependence. These include prices of spot cotton and many securities. Going deeper, I have found in many instances that the observed *R/S* dependence is wholly due to small price changes, which are both more difficult and less worthwhile to arbitrate; large changes are practically *R/S* independent, even though the moments when they occur are highly nonindependent (clustered). But, this and other results of mine leave many issues open. In particular, is or is not the actually observed dependence precisely compatible with efficiency, and why are there so many differences between different series, including cases in which the dependence is negligible?

3. EXCERPTS FROM M 1972c: "STATISTICAL METHODOLOGY FOR NONPERIODIC CYCLES: FROM THE COVARIANCE TO *R/S* ANALYSIS"

This paper begins with a critical and idiosyncratic survey of some of the less known and more dangerous pitfalls of the common statistical methods of time series analysis – namely of the methods using the correlation. It then promotes a new and promising alternative, R/S analysis, and describes a good family of random processes, the fractional noises...

WHILE EXTENDING M 1963b{E14} with more work in the same vein – M 1967j{E15}, M & Taylor 1967{E21} {P.S. 1999: this last paper introduced the notion of "trading time," which became important} – I have also instigated a development perpendicular to the older one. The present paper is meant to be a proselytizing introduction to it. Circa 1960, I was mostly concerned with the *marginal distribution* of various economic time series, *irrespective of their structure of dependence*. Notably, I concentrated on the fact that many economic series, especially price series, are non-Gaussian to the extreme. For this behavior – borrowing from the Bible and from hydrology – I have since coined the self-explanatory term "Noah Effect," M & Wallis 1968{H10}. The equivalent of a flood may be... a price change that some may ride out and others will consider catastrophic.

The well-known "Galton Ogive" distribution has a round head and no tails to speak of. By contrast, the daily changes of the logarithm of the spot price of cotton have a distribution with a pointy head and very long tails. By Gaussian standards, this meant that very small price changes are much too numerous, and large price changes are much too large. I don't see why very large price changes should be handled separately and made a point to examine the observed distributions as wholes. Stared at intensely, they look like a cross between the Gaussian and another classical distribution, Cauchy's, whose density is $(1+x^2)^{-1}\pi^{-1}$. There happens to exist in the literature a whole family of such "hybrids," called L-stable. They fit price change distributions well.

At present, I mostly explore the *structure of long-run dependence in various times series, irrespective of the marginal distribution*. Economic time series tend to be characterized by the presence of clear-cut but not periodic "cycles" of all conceivable "periods," short, medium, and long (that is, "comparable to the length of the total available sample") and where the distinction between "long cycles" and "trends" is very fuzzy. For this behavior I have coined the term "Joseph Effect," M & Wallis 1968{H10}.

While considerable work was invested both in accumulating data in investigating econometric models that generate "cyclic-looking" artificial time series, efforts to characterize the structure of actual series have been minimal;... The first step consisted in observing that something "roughly like it" is encountered in oscillatory autoregressive processes. The initial pioneering observation to this effect was naturally based upon intuitive and casual tests. But, for a whole branch of econometrics, those same tests provide unacceptably flimsy foundations. Pioneering remarks, due among others to Adelman 1956 and Granger 1966, have not been followed up. While non-periodic cyclic behavior is both important and peculiar enough to be viewed as a distinct "phenomenon," the available mathematics (both probability and statistics) has not studied it squarely.

The above remarks set the framework of my current search for the following:

(A) Ways of grasping intuitively the concept of non-periodic "cyclic" long run dependence, contrasting it with the two customary patterns, namely short dependence (Markov character) and periodic variation. The differences between the above kinds of dependence are as deep as those in physics between – respectively – liquids, gases and crystals....

(B) Alternative methods of statistical testing and estimation that stress long run dependence, including methods [that are] insensitive to non-Gaussian margins....

(C) Simple one-variable stochastic processes that look like the data and exhibit the same typical R/S behavior. The main tools I have used are various "fractional noises," ...the counterpart of the L-stable processes I injected into economics around 1960.

M 1972c surveys and elaborates the findings concerning those points that are scattered in my earlier papers. I do have some ideas about the causes of the effects to be described, but, my overall approach is characterized as "phenomenological"....

The quickest way to make a new viewpoint be appreciated is unfortunately "negative": to show how it affects the use of old techniques. This is why this paper grew to also include much critical expositions of the... pitfalls of an apparent lack of correlation and of variance time function analysis. This paper shall proceed to range analysis, a significantly modified variant of "high minus low" analysis. Finally, it shall reach R/S.