Applied Economics Letters, 1996, **3**, 701–706

Long-term stochastic dependence in financial prices: evidence from the German stock market

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Received 23 January 1996

A number of authors have argued that financial prices may exhibit hidden long-term dependence. We consider this claim analysing German stock market data. Applying three different concepts for the identification of long memory effects, virtually no evidence of such behaviour is found for stock market returns. Another recent assertion says that long term memory may not be pertinent to stock returns but rather to the conditional volatility of financial market prices. As it turns out, this claim is very much supported by our investigation of German stock market data. Furthermore, the long memory property is more pronounced in absolute values of returns than in the squares of returns (both used as proxies for volatility). The methods employed are: the time-honoured procedure of estimating the Hurst exponent for the scaling behaviour of the range of cumulative departures from the mean of a time series, the modified range analysis.

I. INTRODUCTION

Stochastic processes with long-term dependence and methods of their identification have been introduced first in the context of hydrology and geophysics (cf Hurst, 1951; Mandelbrot and Wallis, 1969a). Exploring various geophysical records such as river discharges and rainfall data, these authors found typical deviations from a random walk in the data pointing to long range dependence in the generating processes. Hurst proposed a method for the quantification of long-term memory which is based on estimating a parameter for the scaling behaviour of the range of partial sums of the variable under consideration. This method has been refined by Mandelbrot whose influence stimulated similar research in economics. As far as financial time series are concerned, a number of applications of rescaled range analysis as suggested by Mandelbrot can be found during the late 1970s and 1980s (e.g. Greene and Fielitz, 1977; Booth *et al*. 1982; Kaen and Rosenman, 1986). More recently, more refined techniques have been developed to identify long-term dependence such as the modified rescaled range analysis (Lo, 1991) and periodogram regression (Geweke and Porter-Hudak, 1983). All these are non-parametric or semi-parametric approaches and can be applied without detailed assumptions on the structure of the underlying model.

The assertion of long-term dependence in macroeconomic data may not seem implausible at a first view and squares well with theoretical developments focusing on 'hysteresis' in macroeconomic variables such as employment. As far as financial markets are concerned, however, the very notion of dependence (be it short term or long term) is in contradiction to the perceived wisdom of efficient market performance. Since there is hardly any significant autocorrelation between adjacent time periods in stock returns (denoted r_t in the following), it appears even more improbable to find dependence extending over long horizons. However, an appropriate combination of, say, negative short-term dependence and positive long-term dependence may in fact obscure the underlying pattern of some process leading to apparently insignificant autocorrelation despite temporal dependence. On the other hand, the literature on mean reversion in financial prices implicitly assumes the existence of some mechanism which works over long time horizons. Furthermore, we know that squared returns (r_t^2) usually exhibit significant autocorrelation. In time series analyses, this fact has led to the development of the ARCH type stochastic models. However, ARCH and its variants are models incorporating only short-term dependence in variances. It has been argued that this modelling device may not be entirely adequate and that the conditional variances of financial prices may be more accurately characterized by processes incorporating long-term stochastic dependence (Crato and de Lima, 1994). This argument has been extended by Ding *et al*. (1993), who showed that a broad range of power transformations of absolute returns exhibit long autocorrelations and that this property is strongest for power around 1, i.e. for $|r_t|$.[1]

Thus it appears worthwhile to formally test for the presence of long memory in the data and to check whether the results for smaller markets coincide with well-known findings for the US and UK stock markets. In this paper, we analyse the German share price index DAX which is reported daily by the Frankfurt Stock Exchange. We use a time series starting 4 January 1988 and extending to 16 October 1995 (yielding 1949 entries for daily returns). This series covers the entire period from the beginning of daily publication of the DAX until the time when this study was started. Since share indexes are aggregates and it has been demonstrated by Granger (1980) that long memory in indexes may result from aggregation of suitable short-memory elements we also applied our statistical procedures to the time series of those 29 stocks which were included in the index in 1995 and had been on the market already in 1988 (only one of the 30 parts of the DAX as of 1995 had to be neglected because of too short a record). In addition, we also analysed a longer series of reconstructed historical figures[2] of the DAX extending back to 1959 (yielding a time series of 9011 daily entries from 1959 to 1995).

II. METHODS

Our investigation starts by estimating the so-call Hurst exponent for log price changes (returns) as well as squared returns and absolute returns. The Hurst exponent H characterizes the scaling behaviour of the range of cumulative departures of a time series from its mean. Formally, the range R of a time series $\{x_t\}, t = 1, \ldots, T$ is defined as:

$$R_T = \max_{1 \le t \le T} \sum_{t=1}^{T} (x_t - \bar{x}) - \min_{1 \le t \le T} \sum_{t=1}^{T} (x_t - \bar{x}) \quad (1)$$

Here, $\bar{x}$ is the standard estimate of the mean. Usually the range is rescaled by the sample standard deviation (S), yielding the famous R/S statistic. It is well-known that for IID processes the so defined rescaled range increases asymptotically with the square root of observations. However, Hurst and Mandelbrot found a scaling behaviour with exponent $H > 0.5$ to be characteristic of many geophysical time series. Such scaling reflects a tendency of deviations from the mean to be reinforcing and is also characteristic of time series models known as fractional Gaussian noises (cf Mandelbrot and Wallis, 1969b) and fractionally integrated ARMA models (cf. Granger, 1980). In these processes, long-term dependence shows up in a slow (hyperbolic) decay of the autocorrelation function.[3] Relying on the asymptotic scaling relationship.

$$(R/S)_t \sim at^H \quad (2)$$

The Hurst exponent H is usually estimated by a simple linear regression over a sample of increasing time horizons $(s = t_1, t_2, \ldots, T)$:

$$\ln(R/S)_s = \ln(a) + H \ln(s) \quad (3)$$

Though this approach found wide applications in diverse fields, it turned out that no asymptotic distribution theory could be derived for H itself. Hence, no explicit hypothesis testing can be performed and the significance of point estimates $H > 0.5$ or $H < 0.5$ rests on subjective assessment.[4] Luckily, the asymptotic distribution of the rescaled range itself under a composite null hypothesis excluding long-memory could be established by Lo (1991). Using this distribution function and the significance levels reported in his paper, one can test for the significance of apparent traces of long-term memory as indicated by $H \neq 0.5$. However, Lo also showed that the distributional properties of the rescaled range are affected by the presence of short-term memory and he devised a modified rescaled range Q_τ which adjusts for possible short-memory effects by applying the Newey–West heteroscedasticity and autocorrelation consistent estimator in place of the sample standard deviation S:

$$Q_\tau = \frac{1}{S_\tau}\left[\max_{1 \le t \le T} \sum_{t=1}^{T} (x_t - \bar{x}) - \min_{1 \le t \le T} \sum_{t=1}^{T} (x_t - \bar{x})\right] \quad (4)$$

$$S_\tau^2 = S^2 + \frac{2}{T}\sum_{j=1}^{\tau} \omega_j(\tau)\left\{\sum_{i=j+1}^{T} (x_i - \bar{x})(x_{i-j} - \bar{x})\right\},$$
$$\omega_j(\tau) = 1 - \frac{j}{\tau + 1}$$

Under the null of no long term memory the distribution of the random variable $V_\tau = T^{-0.5} Q_\tau$ converges to that of the range of a so-called Brownian bridge. Critical values of this distribution are tabulated in Lo (1991, Table II).

[1]For the UK stock market, this finding of persistence in $|r_t|$ and r_t^2 has been disputed in a recent paper by Brookfield (1995). However, in contrast to Ding *et al.*, who use a very long record of daily data, Brookfield's analysis is performed with a much shorter series of monthly returns. according our own experiences with different sample sizes reported below it seems possible that Brookfield's negative results are at least in part due to the unreliability of the Lo (1991) statistic for small samples and comparatively large truncation lags.

[2]This series was compelled by a working group at the Frankfurt Stock Exchange and is available from this source.

[3]Scaling behaviour with $H > 0.5$ points to anti-persistence in the time series. Models generating this kind of pattern are included in the classes of models mentioned in the main text.

[4]However, in a recent paper, Brooks (1995) performed significance tests for H relying on boot strapping of surrogate data (data with the same distributional characteristics and short-run autocorrelation structure as the time series under consideration).

Table 1. *Estimates of Hurst exponent*

	r_t	r_t^2	$\lvert r_t \rvert$
DAX, 1988–95	0.54	0.72	0.82
DAX, 1959–95	0.55	0.77	0.84
Mean of 29 stocks	0.55	0.70	0.78
Max. of 29 stocks	0.62	0.77	0.88
Min. of 29 stocks	0.49	0.62	0.70

Notes: Following e.g. Mandelbrot and Wallis (1969a) a selection of time steps has been used in the regression 2: specifically, for the shorter series we used $s = 50, 60, \ldots, 100, 200, \ldots, 1900$, whereas for the longer series $s = 50, 60, \ldots, 100, 200, \ldots, 1000, 2000, \ldots, 9000$ was used. $\ln(R/S)$ was then calculated as the mean of a fixed number of non-overlapping intervals for the smaller time steps. For longer time steps, the number of intervals had to be reduced successively and some overlap had to be accepted.

The third method relies on the properties of the periodogram of long-memory processes at low frequencies. This limiting behaviour is derived for the ARFIMA class of long-memory models by Geweke and Porter-Hudak (1983) but since they demonstrate equivalence between the concepts of fractionally integrated ARMA models and fractional Gaussian noises it also extends to the latter. ARFIMA models are generalizations of standard (short-memory) ARMA models:

$$\Phi(B)(1-B)^d y_t = \Theta(B)\epsilon_t \tag{5}$$

where B is the backward-shift operator, and $\Phi(B)$ and $\Theta(B)$ are the AR and MA polynomials respectively. Allowing d to assume non-integer values introduces the possibility of fractional differencing. In fact, it has been found that models with non-integer d display the slow (hyperbolic) decay of the autocorrelations usually identified with long-memory. Geweke and Porter-Hudak show that the spectral density function of a fractional Gaussian noise with Hurst exponent H is identical with that of an ARFIMA model with differencing parameter $d = H - 0.5$. Furthermore, they demonstrate that d can be estimated by a simple linear regression of the log-periodogram $\ln\{I(\lambda_{j\,T})\}$ on $\ln\{4\sin^2(\lambda_{j\,T/2})\}$ at low Fourier frequencies $\lambda_{j\,T} = 2\pi j/T$, i.e.:

$$\begin{aligned} &\ln\{I(\lambda_{j\,T})\} = c - \hat{d}\ln\{4\sin^2(\lambda_{j\,T}/2)\} + v_j \\ &j = 1, 2, \ldots, m < T \end{aligned} \tag{6}$$

where the disturbance v_j can be shown to be asymptotically normal with variance $\pi^2/6$ under normality of the innovation ϵ_t in Equation 5.

Table 2. *Rescaled range test statistics*

		$V_{T=0}$	$V_{T=5}$	$V_{T=10}$	$V_{T=25}$	$V_{T=50}$	$V_{T=100}$	$V_{T=200}$
	DAX, 1988–95	1.13	1.14	1.18	1.18	1.19	1.19	1.28
	DAX, 1959–95	1.33	1.29	1.28	1.22	1.20	1.17	1.16
r_t	29 stocks:							
	significant cases at 90%	1	0	0	0	0	0	0
	significant cases at 95%	0	0	0	0	0	0	0
	significant cases at 99%	0	0	0	0	0	0	0
	DAX, 1988–95	2.73***	2.28***	2.15***	1.96**	1.77*	1.59	1.46
	DAX, 1959–95	5.35***	3.94***	3.49***	2.88***	2.51***	2.18***	1.92**
r_t^2	29 stocks:							
	significant cases at 90%	29	23	3	19	12	2	0
	significant cases at 95%	27	22	20	16	7	0	0
	significant cases at 99%	24	18	15	6	1	0	0
	DAX, 1988–95	4.14***	3.10***	2.68***	2.11***	1.71	1.41	1.22
	DAX, 1959–95	7.89***	5.44***	4.58***	3.52***	2.87***	2.37***	1.95**
$\lvert r_t \rvert$	29 stocks:							
	significant cases at 90%	29	29	29	25	13	3	0
	significant cases at 95%	29	29	28	20	11	0	0
	significant cases at 99%	29	27	22	11	4	0	0

Notes: *, **, and *** denote significance at the (two-sided) 90%, 95% and 99% level, respectively. The 90%, 95% and 99% intervals are given by {0.861, 1.747}, {0.809, 1.862}, and {0.721, 2.098}, respectively, cf. Lo (1991), Table II.

Table 3. *Estimates of fractional differencing parameter d from periodogram regression*

		$T^{0.55}$	$T^{0.50}$	$T^{0.45}$
	DAX, 1988–95	0.06	0.11	−0.03
	DAX, 1959–95	0.03	0.00	0.02
	Mean (29 stocks)	0.03	0.02	−0.02
r_t	Min (29 stocks)	−0.13	−0.24	−0.31
	Max (29 stocks)	0.22	0.26	0.21
	significant cases at 90%	2	5	2
	significant cases at 95%	1	3	2
	significant cases at 99%	0	0	0
	DAX, 1988–95	0.18**	0.27**	0.21
	DAX, 1959–95	0.25***	0.24***	0.24**
	Mean (29 stocks)	0.17	0.23	0.24
r_t^2	Min (29 stocks)	0.01	0.00	0.00
	Max (29 stocks)	0.29	0.46	0.54
	significant cases at 90%	19	20	15
	significant cases at 95%	18	16	13
	significant cases at 99%	8	10	5
	DAX, 1988–95	0.38***	0.59***	0.45***
	DAX, 1959–95	0.37***	0.37***	0.41***
	Mean (29 stocks)	0.32	0.42	0.41
$\|r_t\|$	Min (29 stocks)	0.19	0.22	0.24
	Max (29 stocks)	0.43	0.60	0.62
	significant cases at 90%	29	29	28
	significant cases at 95%	29	29	27
	significant cases at 99%	27	25	20

Notes: *, **, and *** again denote significance at the (two-sided) 90%, 95% and 99% level, respectively. Inference is based on the asymptotic distribution of the estimate of $d : \hat{d} \sim N\left(d, \frac{\pi^2}{6\sum_{t=1}^{m}(y_t-\bar{y})^2}\right)$ with y_t, the regressor in Equation 6:$y_t = \ln\{4\sin^2(\lambda_{j,T}/2)\}$.

III. Results

Table 1 gives the Hurst exponents for returns, squared returns and absolute returns of the DAX as well as the range of values obtained for the 29 individual share price records. In an attempt to the short-term dependence and pre-asymptotic behaviour we restricted the regression Equation 3 (selected) lags $s \geq 50$. One observes that for the period 1988 to 1995 as well as for the extended historical record the statistics for returns stay close to the benchmark 0.5, which means that there is at least no strong indication of long memory. Squared returns as well as absolute returns are, however, much more indicative of long-term effects with Hurst exponents ranging consistently above one half. Two more features are worth remarking here: First, looking at the detailed results for individual stocks (available upon request) we always find: $H(|r_t|) > H(r_t^2) > H(r_t)$. Hence, confirming the findings by Ding *et al.* (1993), the long-memory property is uniformly stronger in absolute returns than in squared returns. Second, the fit of the regression Equation 3 is always very good yielding R^2's around 0.99 throughout. Hence, the scaling law Equation 2 with estimated exponent H appears to depict very accurately the behaviour of the rescaled range of the time series under consideration.

Which of these results can be judged to characterize statistically significant deviations from $H = 0.5$? Table 2 reports the test statistics $V_\tau = T^{-0.5}Q_\tau$ for lags $\tau = 0$, 5, 10, 25, 50, 100 and 200.[5] The first case corresponds to a test based

[5] We also tried the data-dependent rule for selecting τ given in Lo (1991, p. 1302). Typically, the selected lag length was between 0 and 3 for returns yielding insignificant test statistics, while it ranged from 3 to 12 for squared and absolute entries yielding highly significant values of V_τ.

on the unadjusted rescaled range R/S used by Hurst and Mandelbrot while the other cases adjust for the possible presence of short-run autocorrelation according to Equation 4. As can be observed, there is an almost complete lack of evidence of long-term memory in returns. The only, marginally significant case out of 29 stock is one of anti-persistance ; i.e. corresponding to $H < 0.5$, rather than positive long-run effects. Again, the picture is very different for squared and absolute returns: the statistics are highly significant for the index over 1988–95 as well as for the majority of individual shares up to lag $\tau = 25$, but significance 'vanishes' when we take into account 50, 100 or 200 lags in the correction of the variance. However, Lo himself showed by means of Monte Carlo simulation that the power of his test is considerably reduced as the truncation parameter τ is increased as compared with sample size. Hence, we have reason to question the reliability of the statistic for large lags τ. This means that conclusion of the kind that there be short-run dependence over 50 lags would not be warranted. On the other hand, the results for the long time series remain significant at the 95% or even at the 99% level even at those large truncation lags. This finding suggests the conclusion that the insignificant statistics of the shorter series for $\tau \geq 50$ are likely to be due to the smaller sample size.

Table 3 gives the estimates of the differencing parameter d from log-periodogram regression. Here one faces a problem of selecting an appropriate sample of small Fourier frequencies similarly to the problem of selecting appropriate lag length in the Lo statistic. Geweke and Porter-Hudak (1983) as well as a number of other authors recommend choosing a number m of smallest frequencies to be used in the regression Equation 6, where m is the largest integer smaller than $T^{0.5}$. We followed them and tested the sensitivity of the estimates by also calculating point estimates from samples $j \leq m = \text{int}[T^{0.55}]$ and $m = \text{int}[T^{0.45}]$.[6] The results broadly confirm our earlier insights: the estimated differencing parameters d stay close to $d = 0$ (i.e. $H = 0.5$) for returns, whereas they are significantly above zero for squared and absolute returns. Also, we found that absolute returns have higher values of d than squared returns implying slower decay of autocorrelation of the former. Though the results for the index and the means from our 29 stocks are in quite good agreement with the figures obtained with the Hurst exponent (Table 1), the estimates are less uniform among our sample of stocks. For example, while the Hurst exponent for returns ranged within the interval {0.49, 0.62} the differencing parameters obtained using $m = \text{int}[T^{0.5}]$ extend from -0.24 to 0.26. With the above relationship $d = H - 0.5$ one, therefore, obtains the much larger bandwidth {0.26, 0.76} for the variation of the scaling parameter H among stocks.

IV. CONCLUSIONS

Our investigation of German stock market data shows that there is no evidence for (positive or negative) long-term dependence in the returns series. This is in sympathy with findings for the USA (Lo, 1991; Goetzman, 1991) and the UK (Mills, 1993). As has been emphasized recently by Ding *et al.* (1993), the long-memory property is strongest in absolute returns. Another remarkable result is the similarity of the results over different time horizons: estimated Hurst exponents as well as differencing parameters are almost identical for the seven-year period 1988 to 1995 and for the historical record covering 37 years. This suggests that the pattern of dependence in volatility remained quite constant over time.[7] Because of this unanimity, we are also tempted to place more weight on the significant results of Lo's modified rescaled range statistic at large lags $\tau \geq 50$ obtained for the historical data set than at the insignificant results obtained for the smaller record. Finally, strong evidence for long-memory in volatility does not only pertain to the German share price index DAX, but was also obtained for the majority of individual stocks within the index. This suggests that long-term dependence in volatility is not merely a consequence of aggregation (a possibility put forward by Granger, 1980) but has to be traced back to the price formation mechanism of financial markets.

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[6] Künsch (1986) points out that the periodogram at the smallest Fourier frequencies may be affected by small monotonic trends. They, therefore, recommend to exclude a few frequencies close to zero from the regression. We controlled for this source of error by repeating the calculations reported in Table 3 dropping frequencies $j < T^{0.2}$. The results showed no pronounced deviation from the picture available in Table 3, though we obtained fewer significant estimates because of reduced sample size.

[7] Brock and de Lima (1995) have shown that both Lo's R/S statistic as well as the periodogram regression approach tend to spuriously indicate long memory in the squared entries of short-memory regime switching processes, whereas the Hurst exponent estimation appears to give more robust results under such circumstances. The conformity of our results under all three techniques suggests that this potential source of error may not be relevant to our data.

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